Multi-objective Optimization: Joint Inversion of Geophysical Datasets

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Multi-objective Optimization

- The process of optimizing systematically and simultaneously a collection of objective functions is called multi-objective optimization (MOO) or vector optimization.
- The general MOO is defined as follows:

$$Minimize\ F(\mathrm{m}) = [f_1(\mathrm{m}), f_2(\mathrm{m}), f_3(\mathrm{m}), .., fn(\mathrm{m})]$$

Within a feasible regions and with some constraints.

MOO-Common Approach
Weighted Global Criterion Method

$$\text{minimize} \sum_{i=1}^{n} w_i \, fi(\boldsymbol{m})$$

Example 1

- Joint inversion of first-arrival traveltime and gravity data
- Results from OBS data over the subduction zone of Taiwan.

Objective

Joint Inversion of Seismic and Gravity Data

Why Joint Inversion?

Simultaneous interpretation of multi-set geophysical data can be useful to obtain an integrated model which will be more geologically feasible.

Combined use of different methods helps to fill up the gap of one method by the other and thus, one can capitalize on the advantages of each method.

Physical Parameters

- Gravity anomalies are caused by density contrasts.
- Seismic travel times are sensitive to wave velocities that are functions of density.
- Seismic wave amplitudes are affected by variations in V_p, V_s and density.
- Often seismic velocity and density are correlated.

Methodology

Travel time

$\mathbf{T}=f_1(\alpha(\mathbf{x}))$

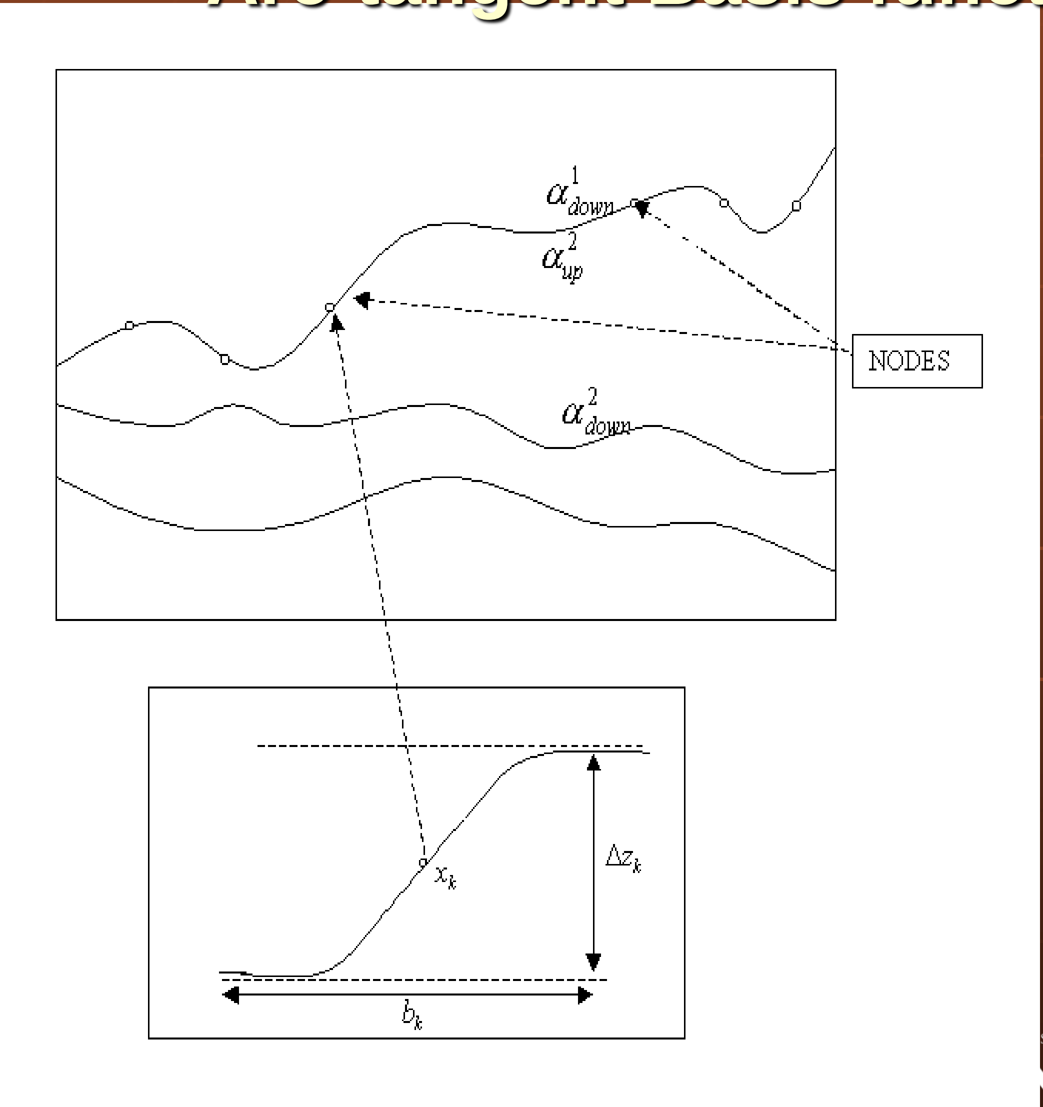
Gravity

$\mathbf{g}= f_2(\rho(\mathbf{x}))$

α(x) represent spatial distributions of compressional wave velocity

ρ(x) represent density and x represent a position vector.

Model Parameterization
Arc-tangent Basis functions



x_k is the horizontal location of an arc-tangent node,

Δz_k is the vertical throw attained asymptotically over a horizontal distance of b_k .

Arc-tangent

$$z(x) = z_0 + \sum_{k=1}^{n} \Delta z_k \left(0.5 + \frac{1}{\pi} \tan^{-1}\left(\frac{x - x_k}{b_k} \right) \right)$$

z is the depth,

n is the number of arc-tangent nodes,

z_0 is the average depth of the interface,

Velocity-Density Relation

$$\rho(x,z)=a_1+a_2\,\alpha(x,z)+a_3\,\alpha^2(x,z)+a_4\,\alpha^3(x,z)+a_5\,\alpha^4(x,z)$$

$$a_1=0.6997,\ a_2=2.23,\ a_3=-0.598,\ a_4=0.0703,\ a_5=-0.00283$$

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Objective function

$$F\left(\alpha(\mathbf{x}),\rho(\mathbf{x})\right)=\left\|\mathbf{T}_{obs}-\mathbf{T}_{syn}\right\|+w\left\|\mathbf{g}_{obs}-\mathbf{g}_{syn}\right\|$$

T_{obs} and T_{syn} - observed and synthetic travel time data,
g_{obs} and g_{syn} - observed and synthetic gravity data,

|| . || - suitable norm

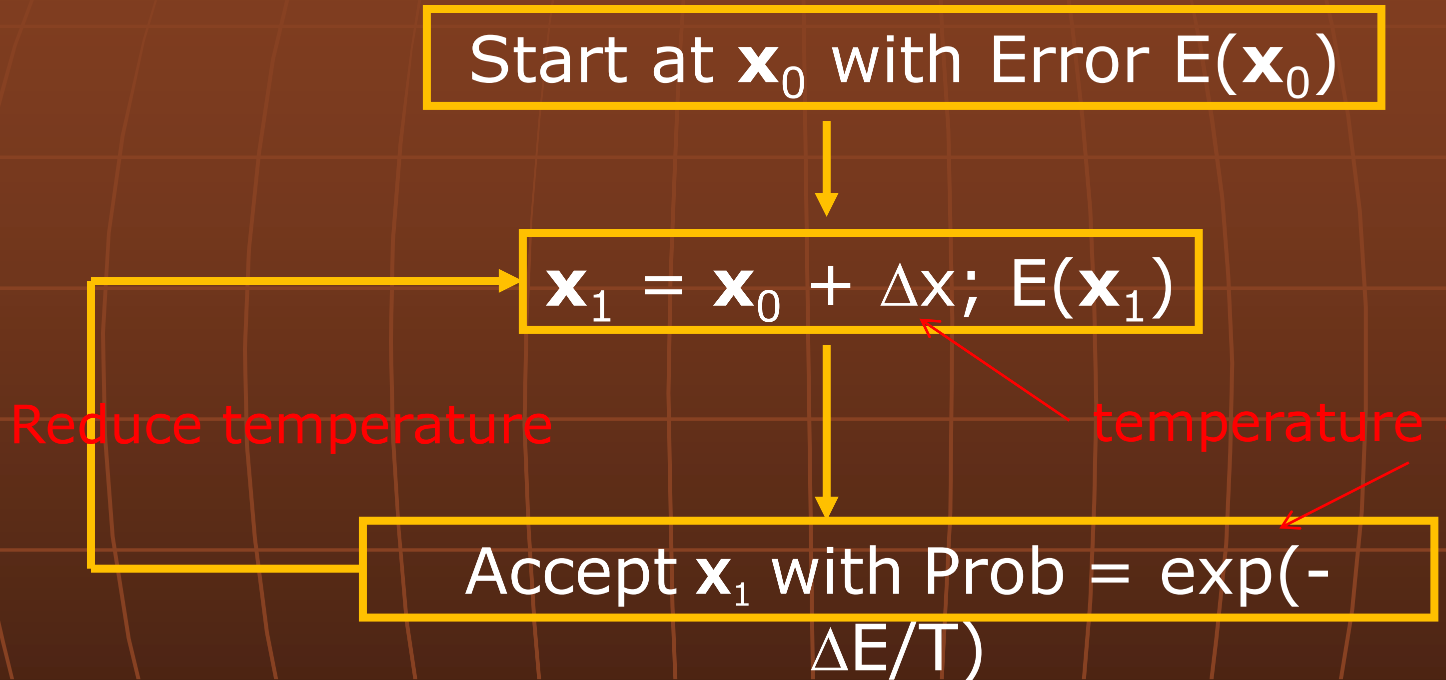
w - weighting factor

$$F\left(\alpha(\mathbf{x}),\rho(\mathbf{x})\right)=\left(\mathbf{T}_{obs}-\mathbf{T}_{syn}\right)^{T}\mathbf{C}_{T}^{-1}\left(\mathbf{T}_{obs}-\mathbf{T}_{syn}\right)+w\left(\mathbf{g}_{obs}-\mathbf{g}_{syn}\right)^{T}\mathbf{C}_{g}^{-1}\left(\mathbf{g}_{obs}-\mathbf{g}_{syn}\right)$$

C_T and C_g are the data covariance matrices for travel time and gravity respectively.

Introduction

Very Fast Simulated Annealing



Weigh each model with exp(-E(x)) and then evaluate the integrals to estimate the marginal PPD, posterior mean, covariance and correlation matrices

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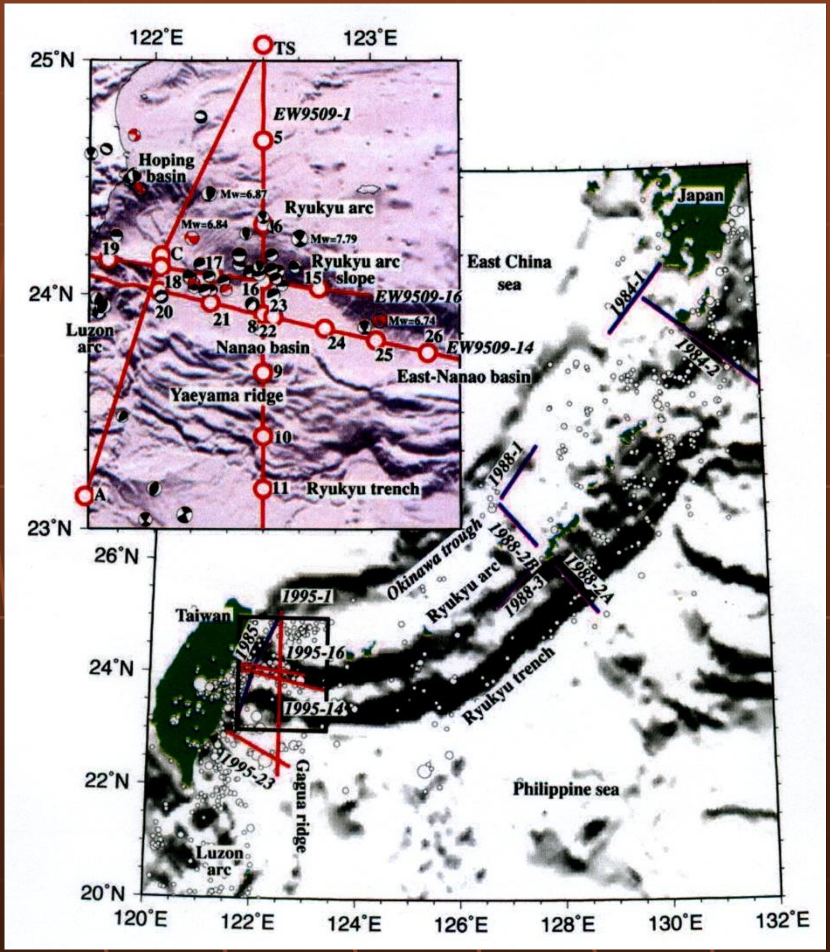
Study Area



Taiwan is basically an artifact of an ongoing collision between the Eurasian plate and Luzon volcanic arc at the southwestern edge of Philippine Sea plate. The collision has been propagating from north to south due to an oblique orientation of the Luzon arc with respect to Eurasian plate generating the Taiwan Orogen.

NW subduction of PSP beneath the Eurasian plate in the NW portion of the Circum-Pacific belt is the main tectonic motion causing high seismicity in the

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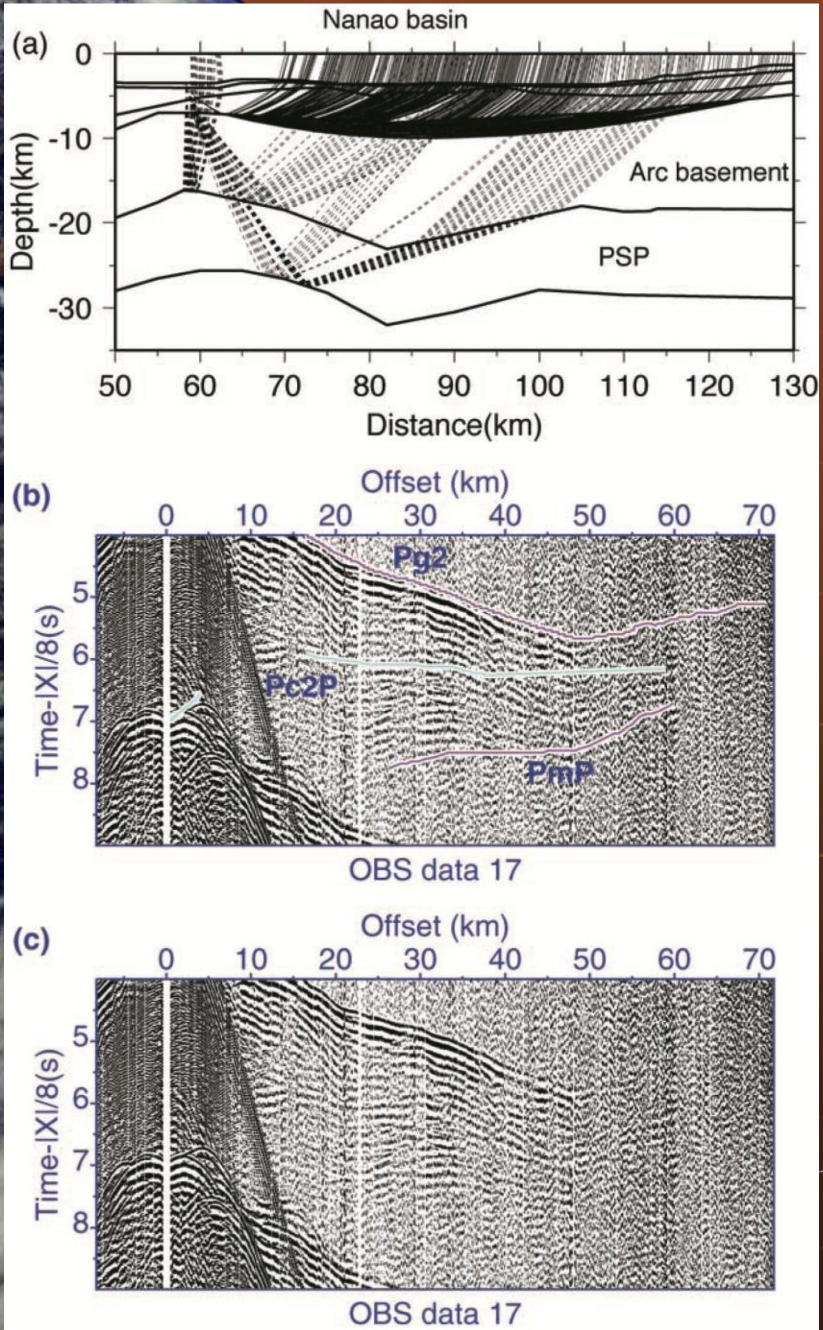

Line 1 OBS 17

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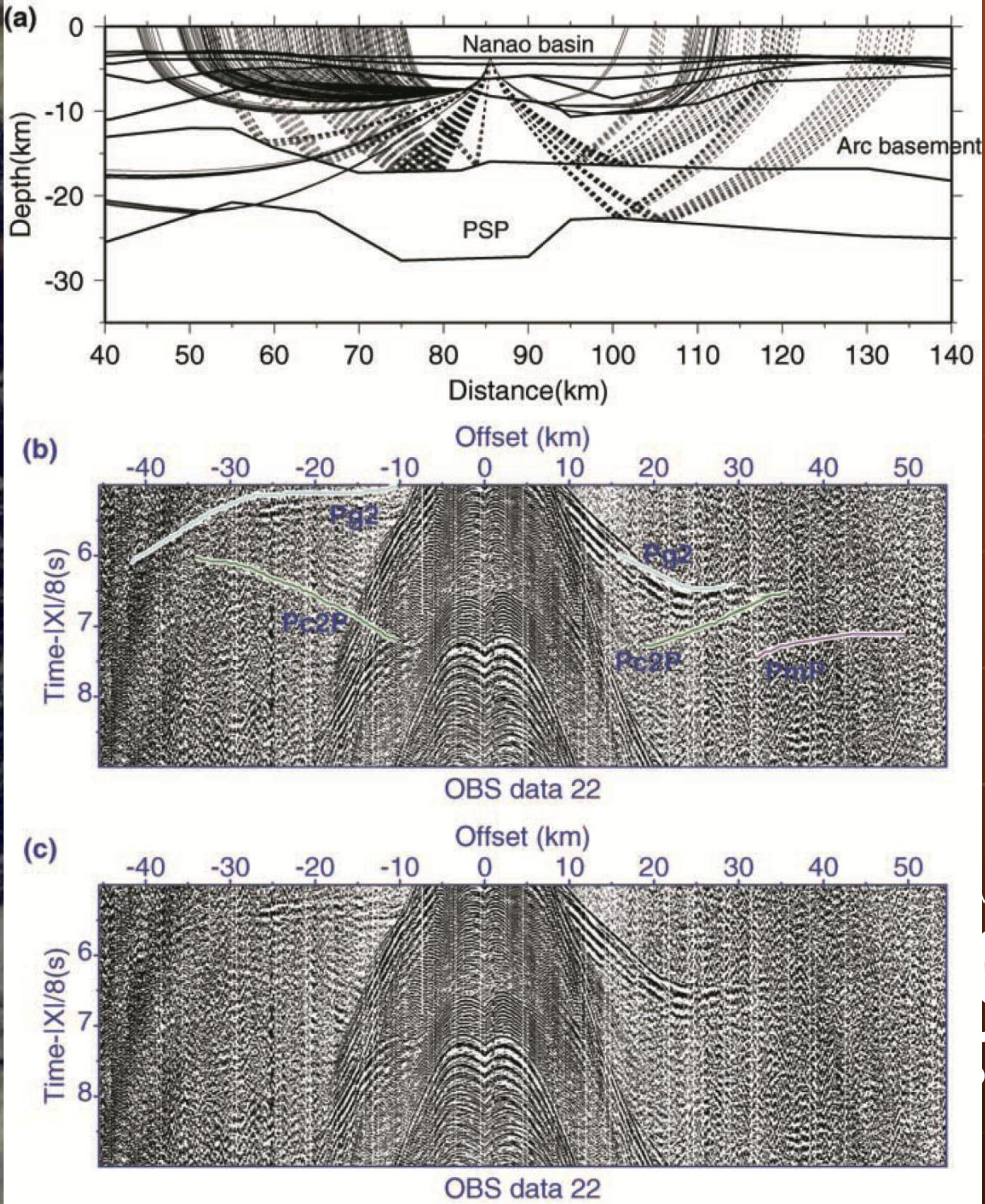
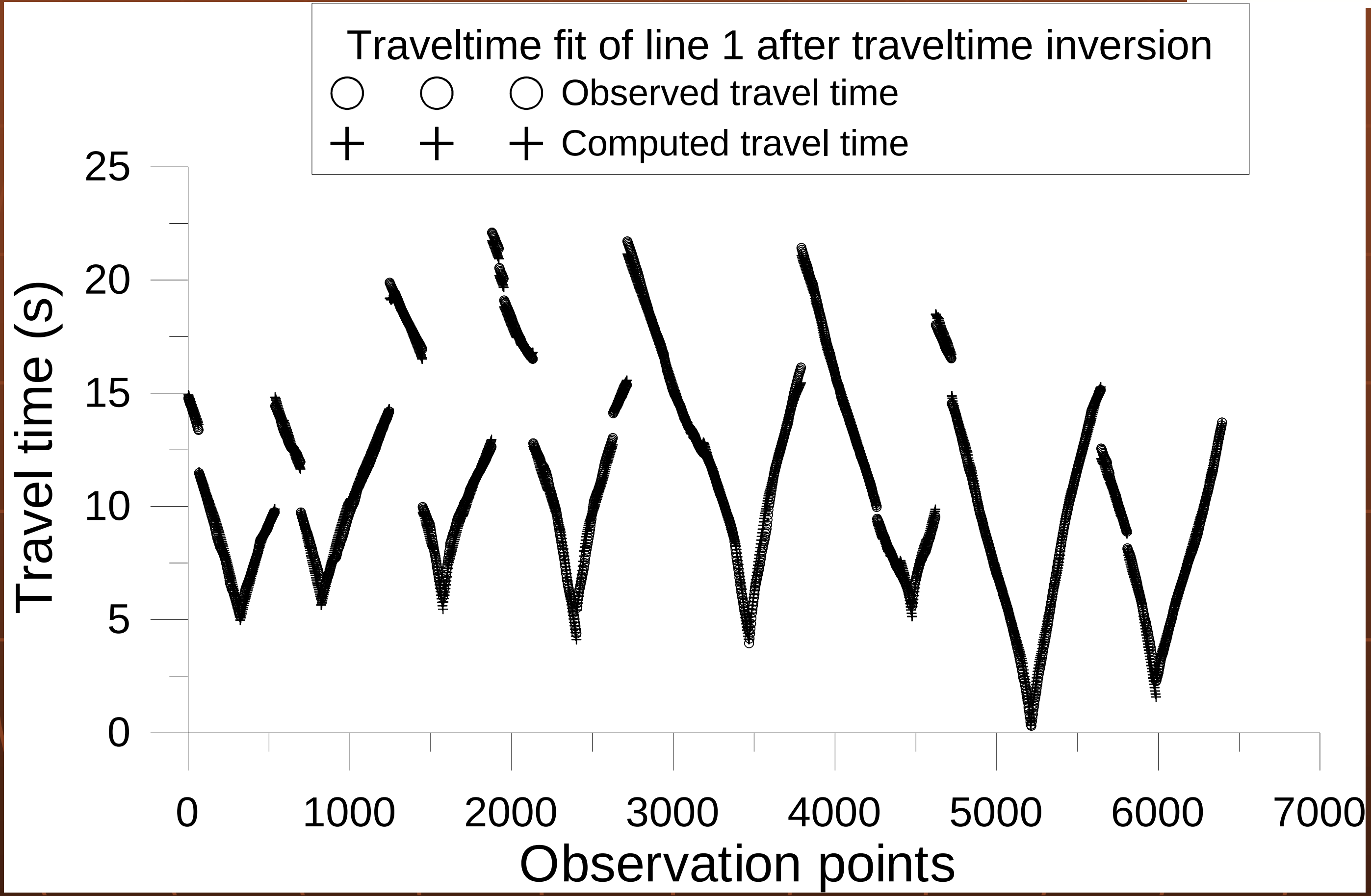


Line 14 OBS 22

Result Line 1



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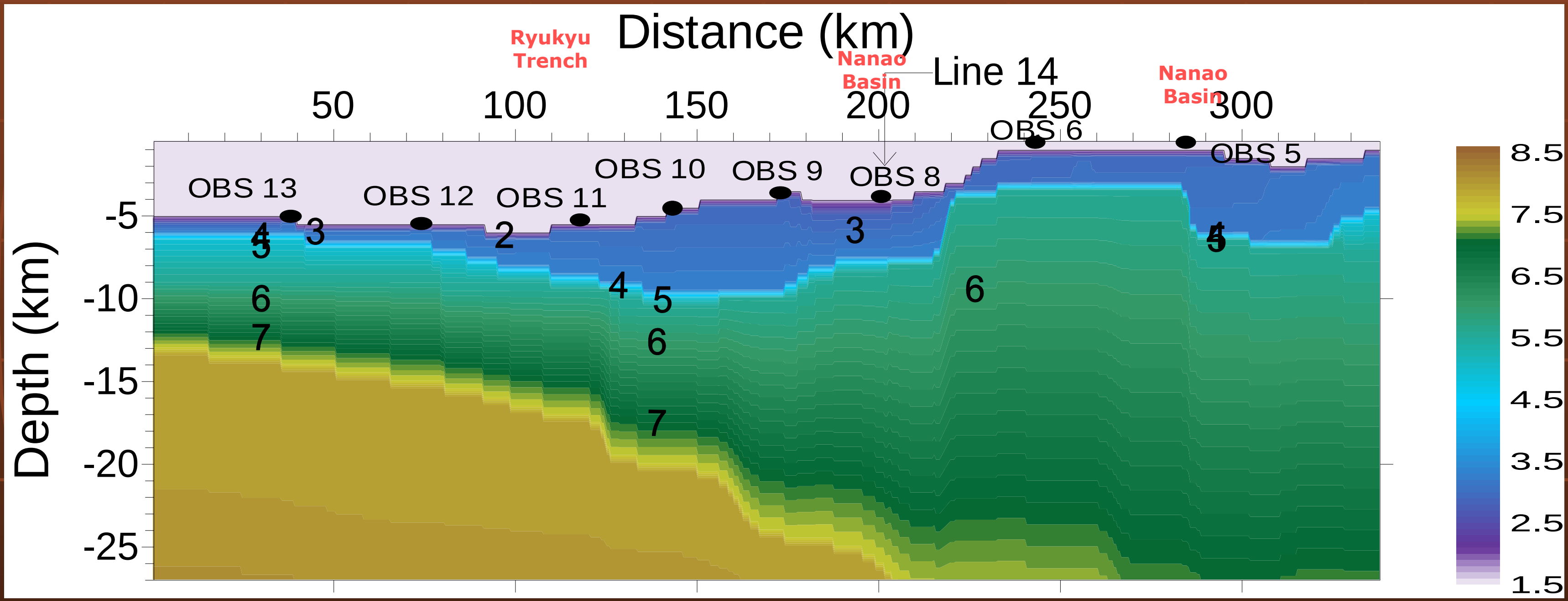
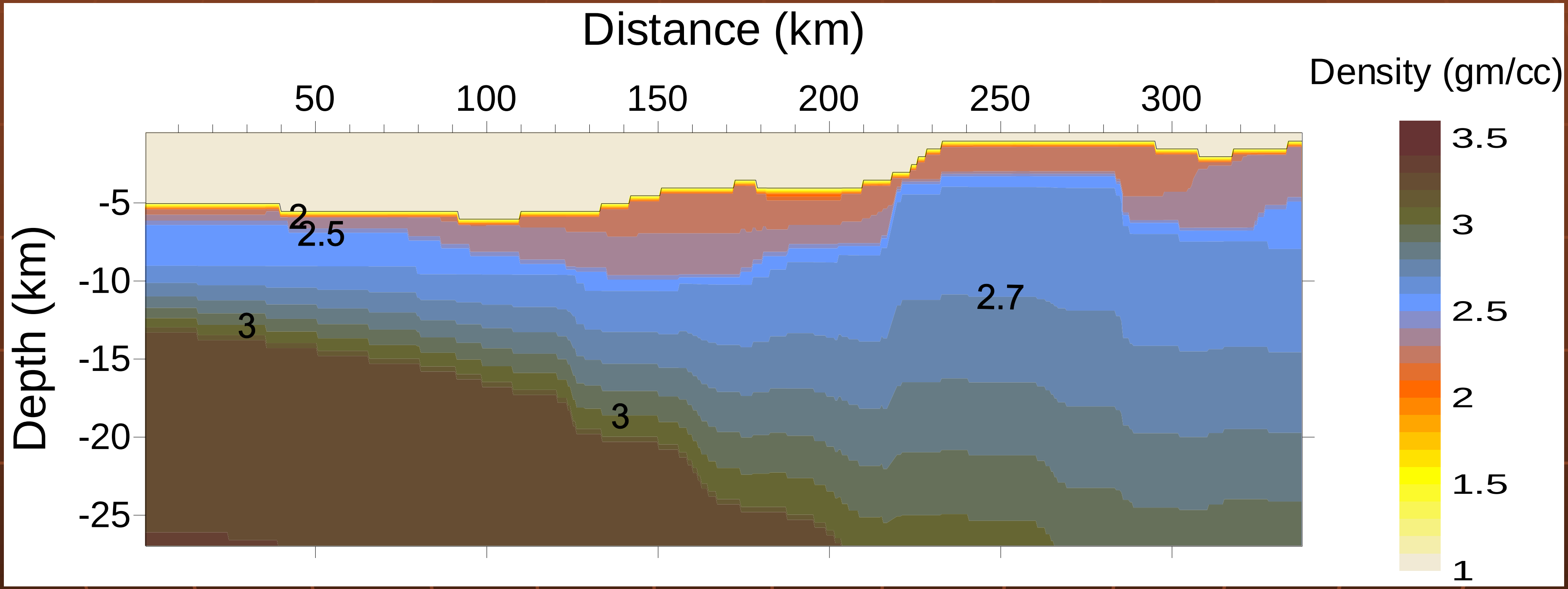
Line 1



Velocity model obtained after travel time inversion

Line 1



Density model obtained from the velocity model of travel time inversion

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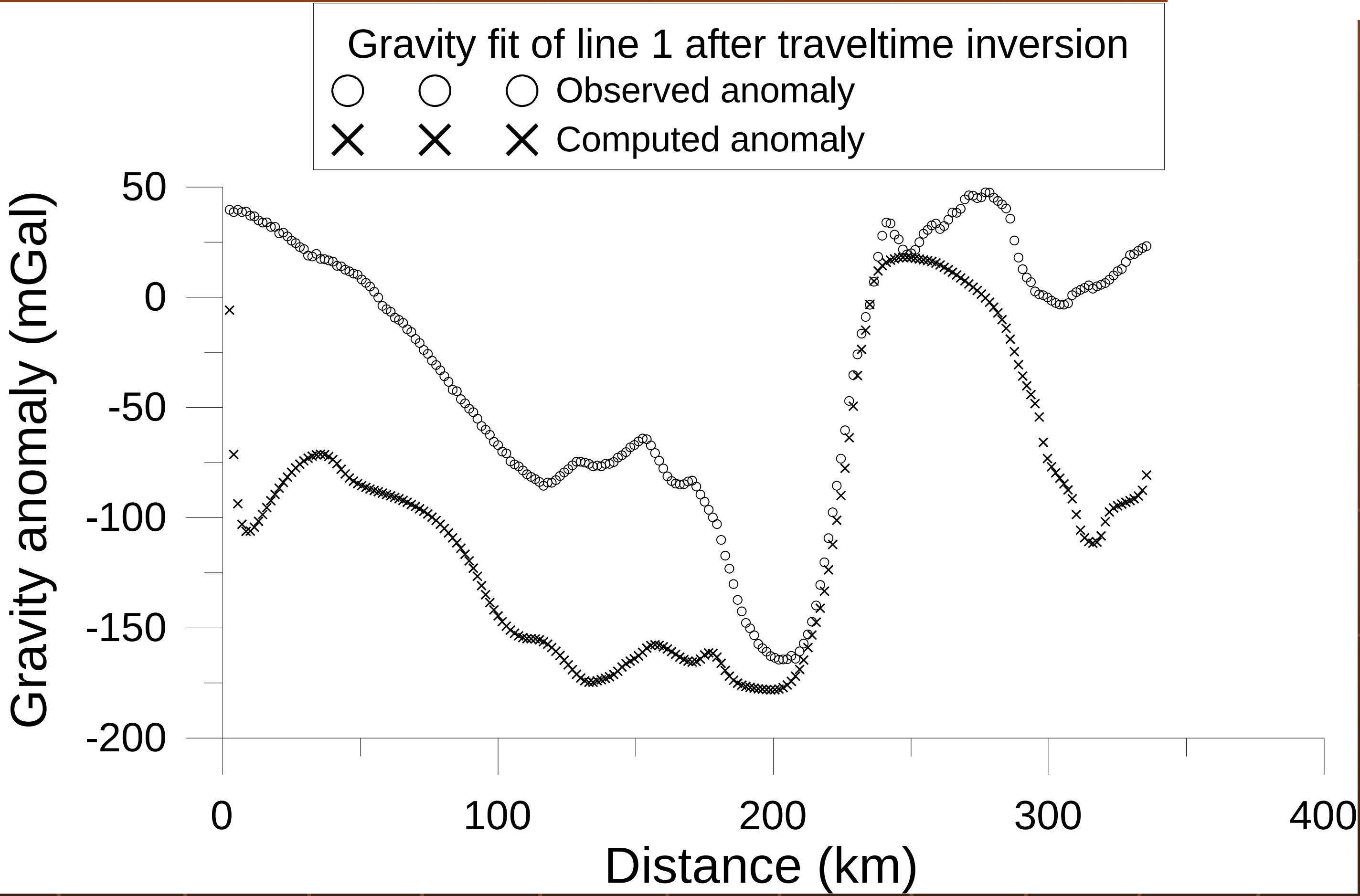
Line 1



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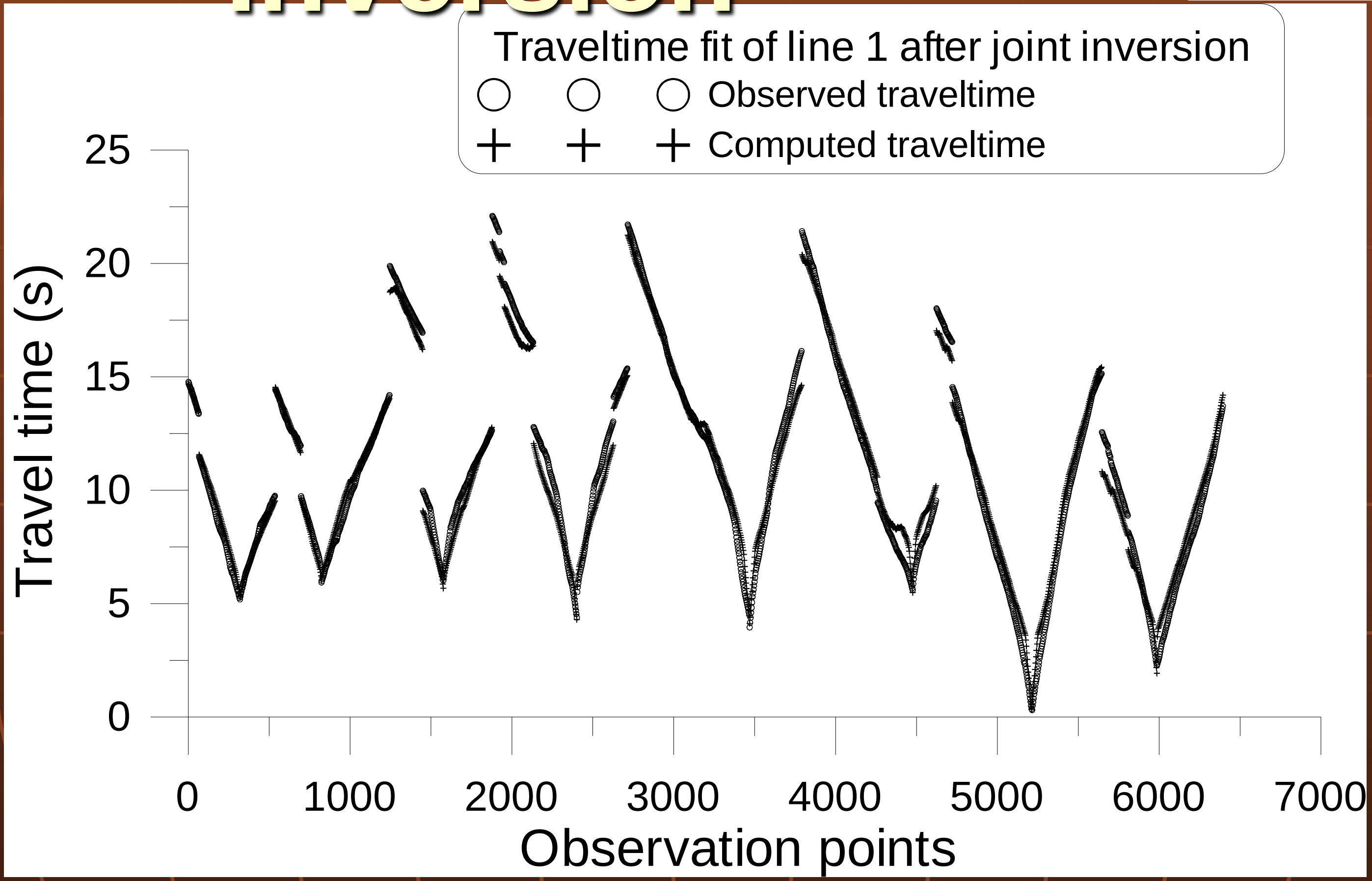
Line 1 – Joint Inversion



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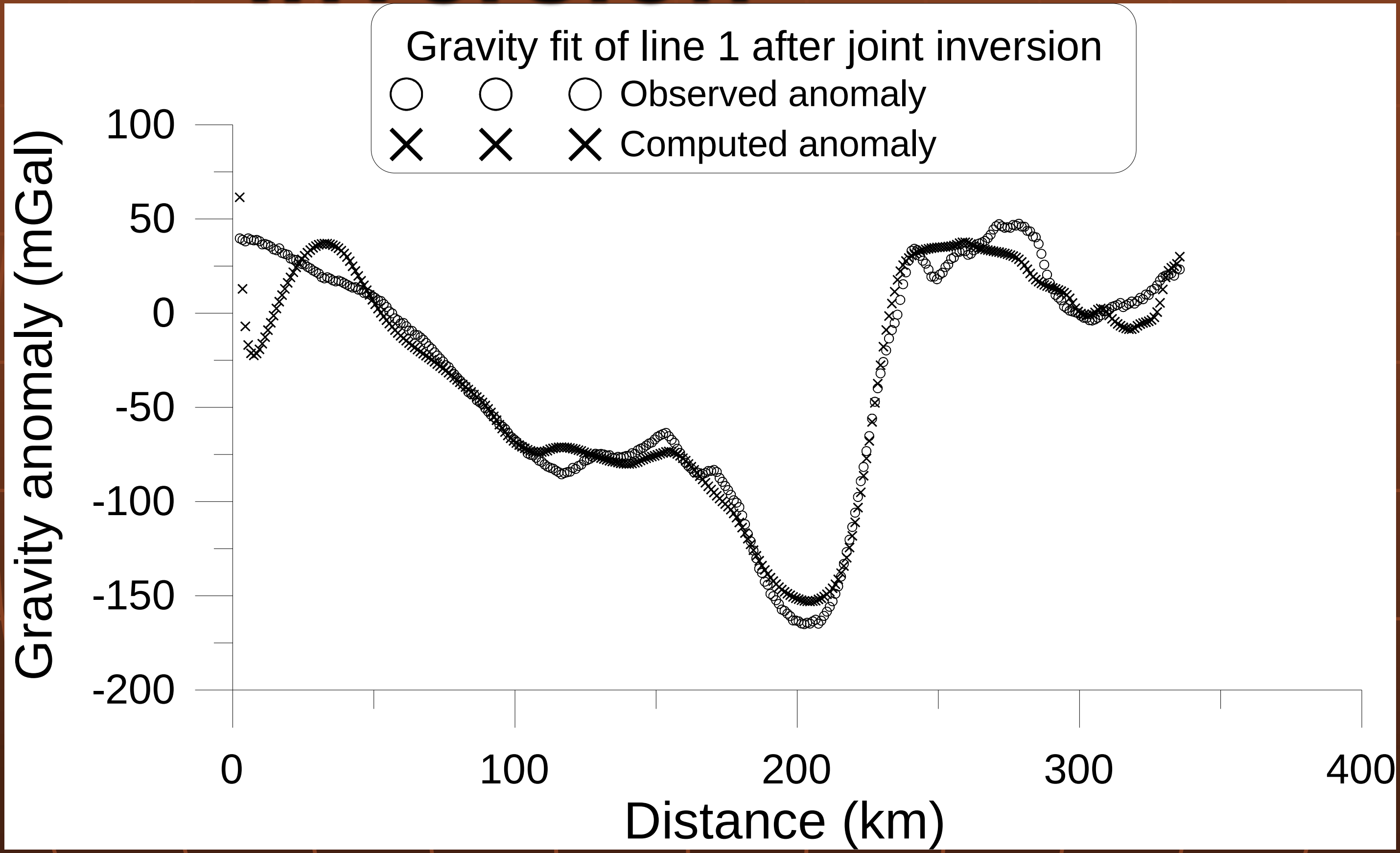
Line 1 – Joint Inversion



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Line 1 – Joint

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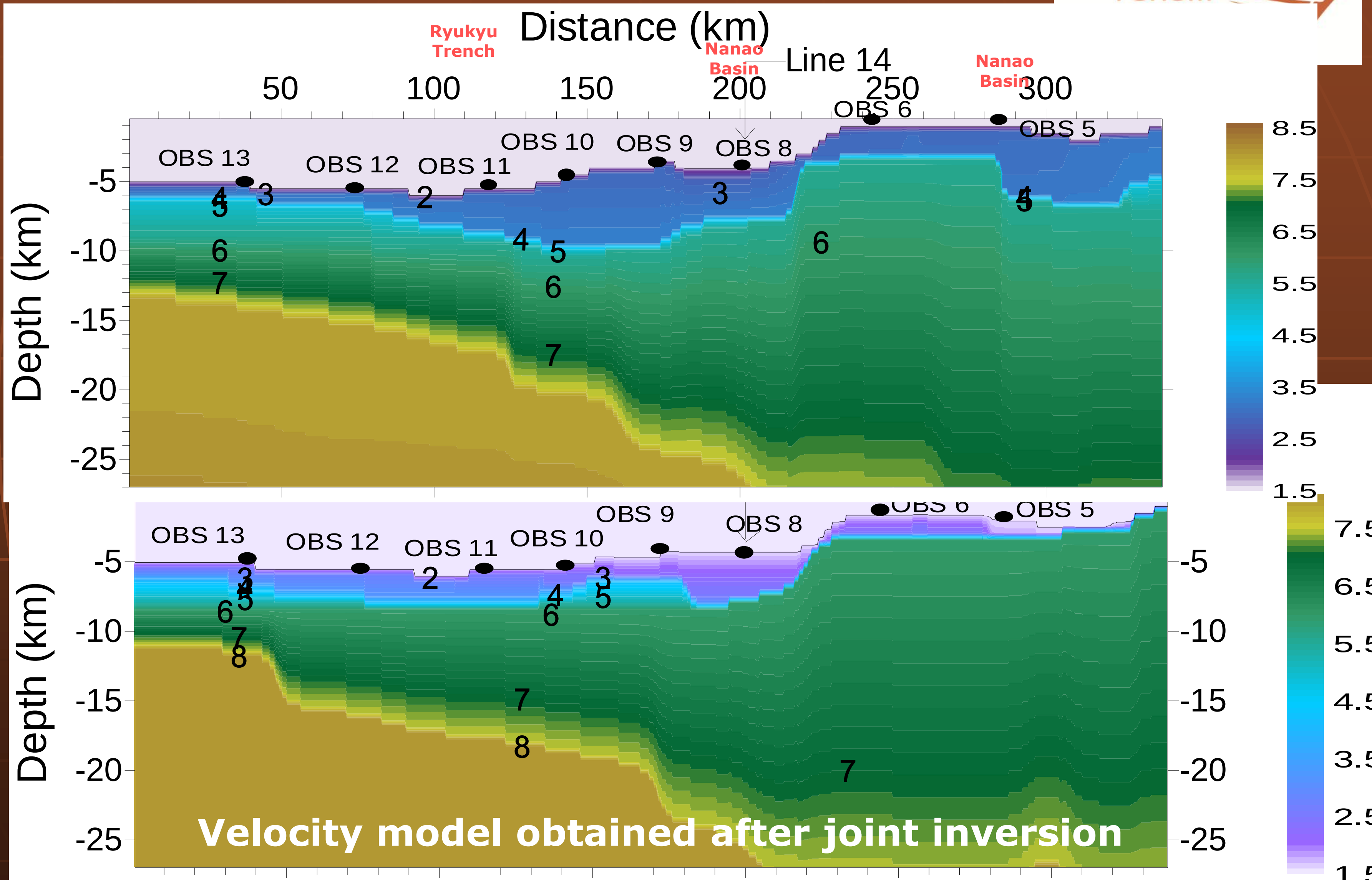

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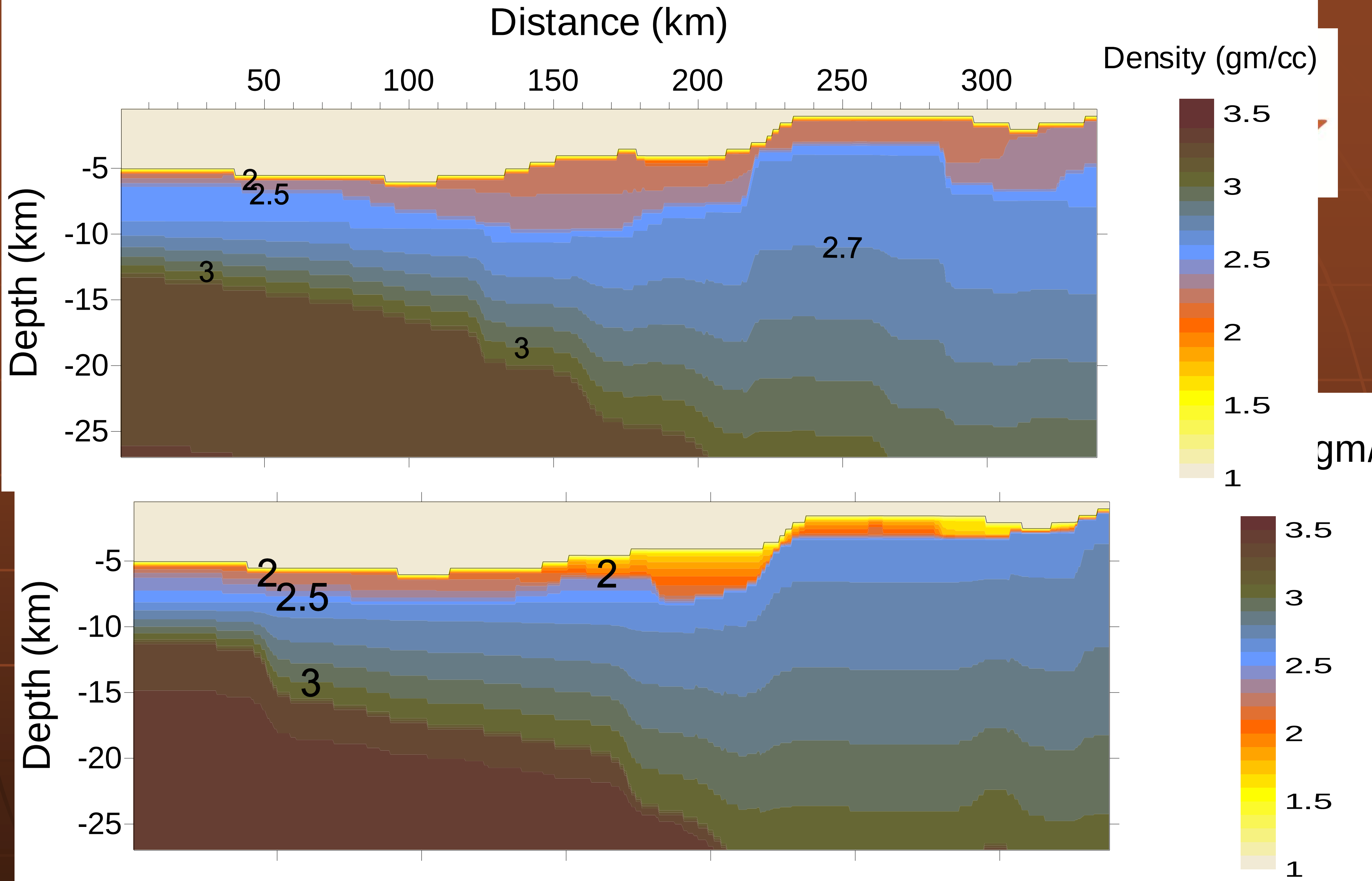
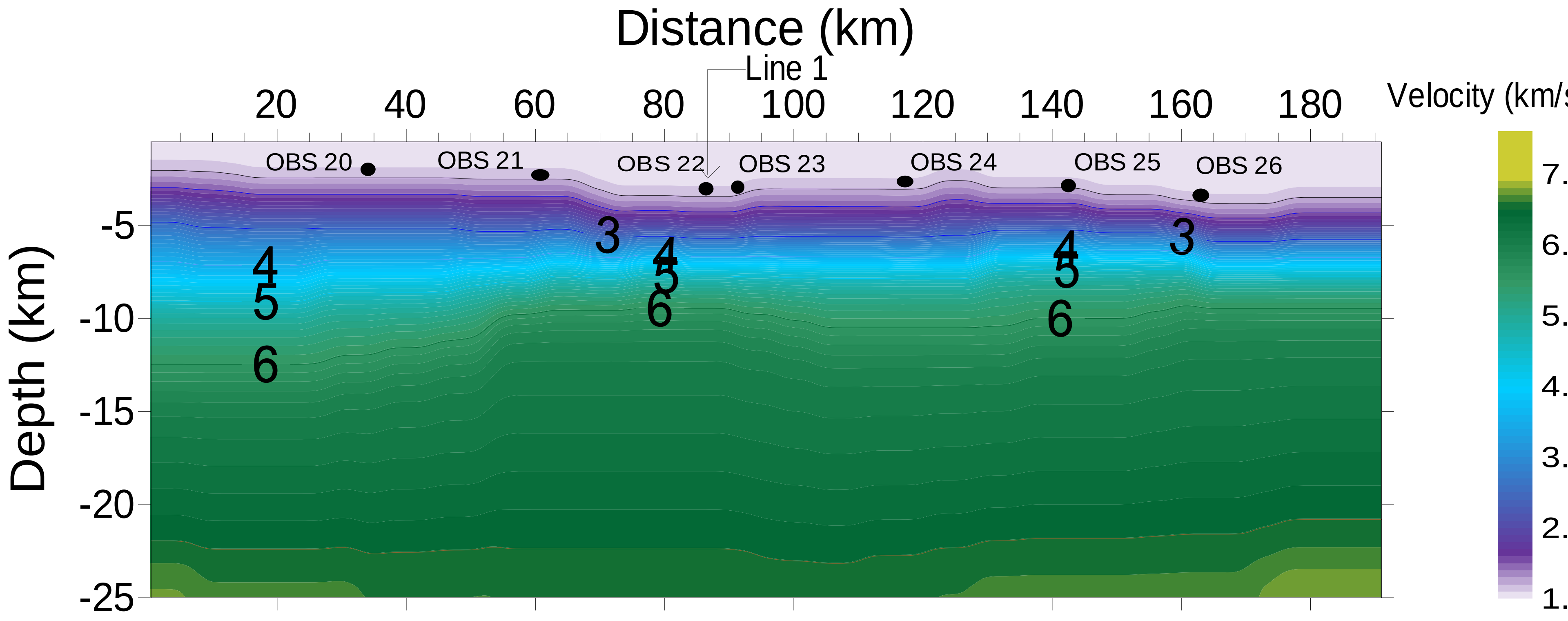
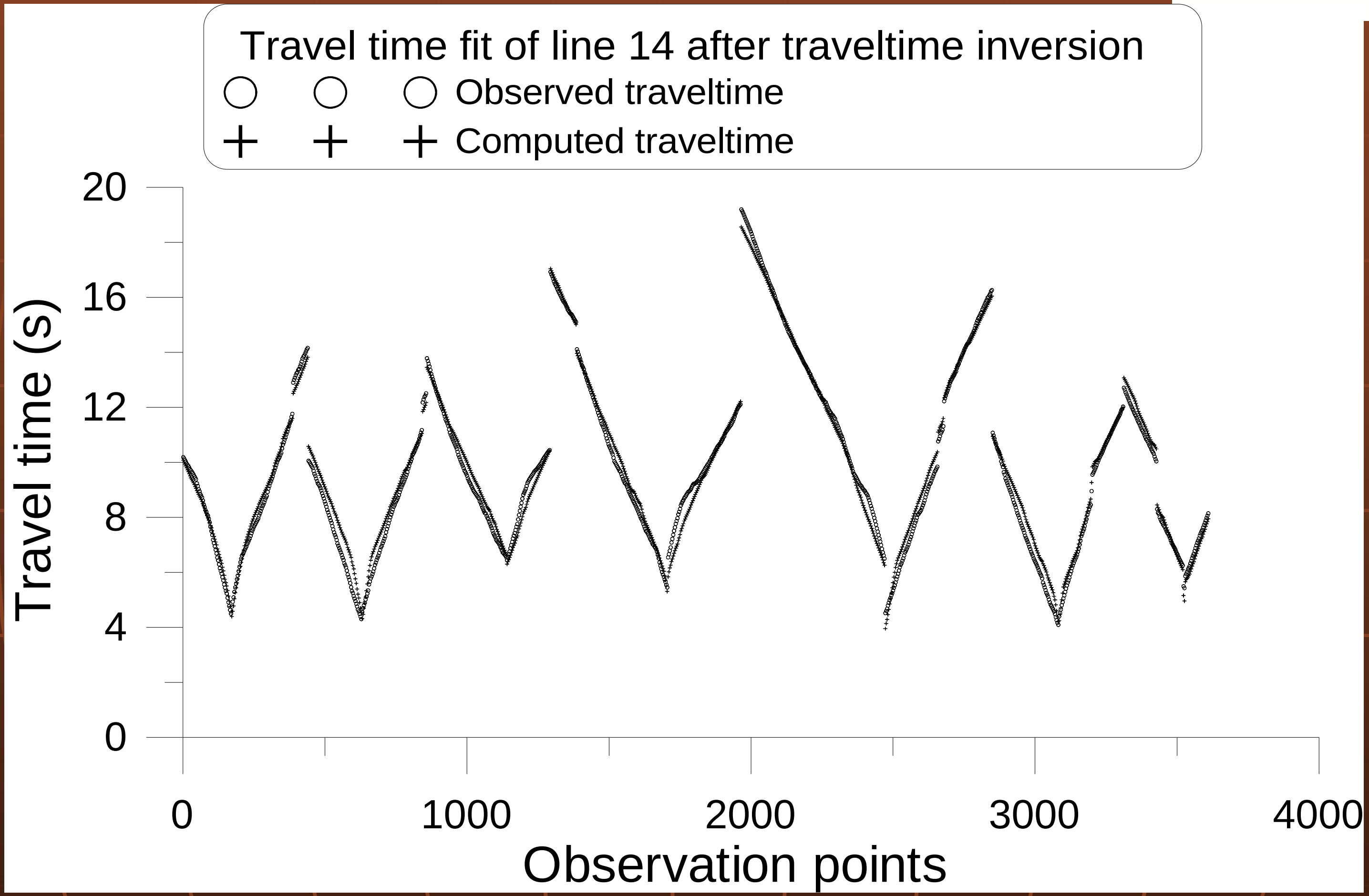


Density model obtained after joint inversion

Line 14



velocity model obtained travel time inversion



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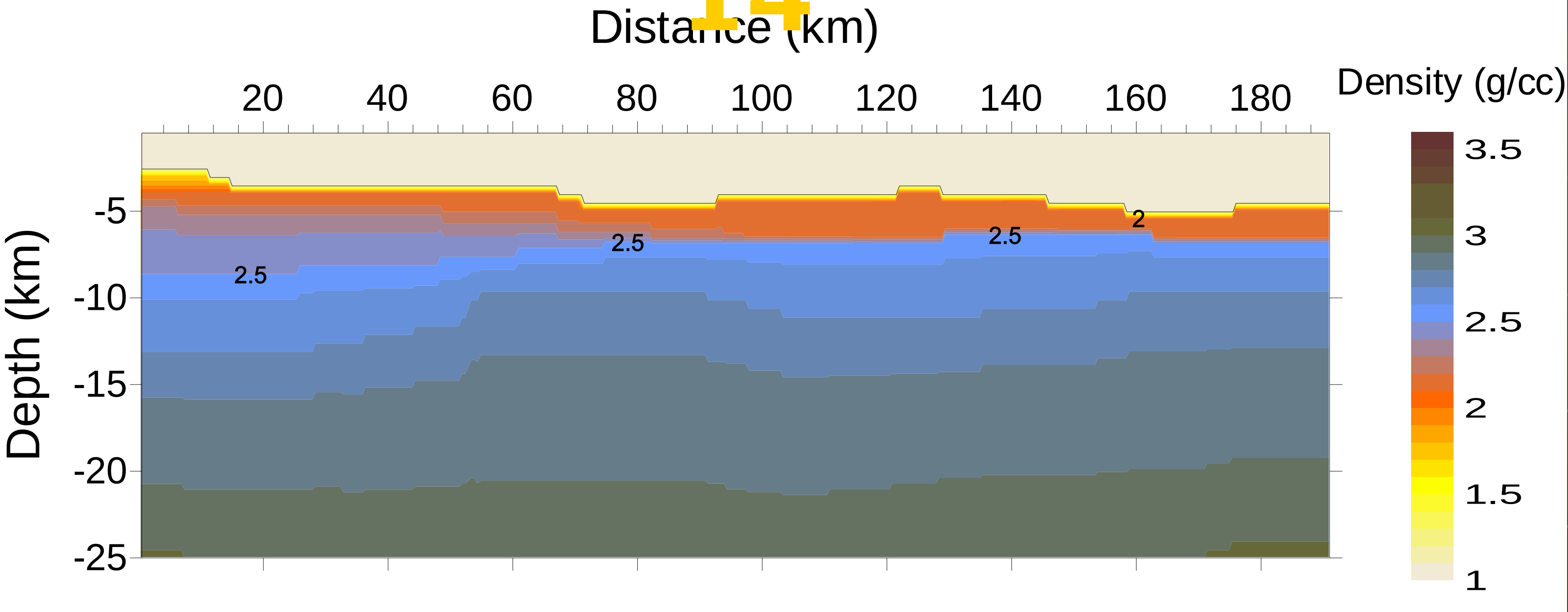
Line 14



Density model obtained after travel time inversion

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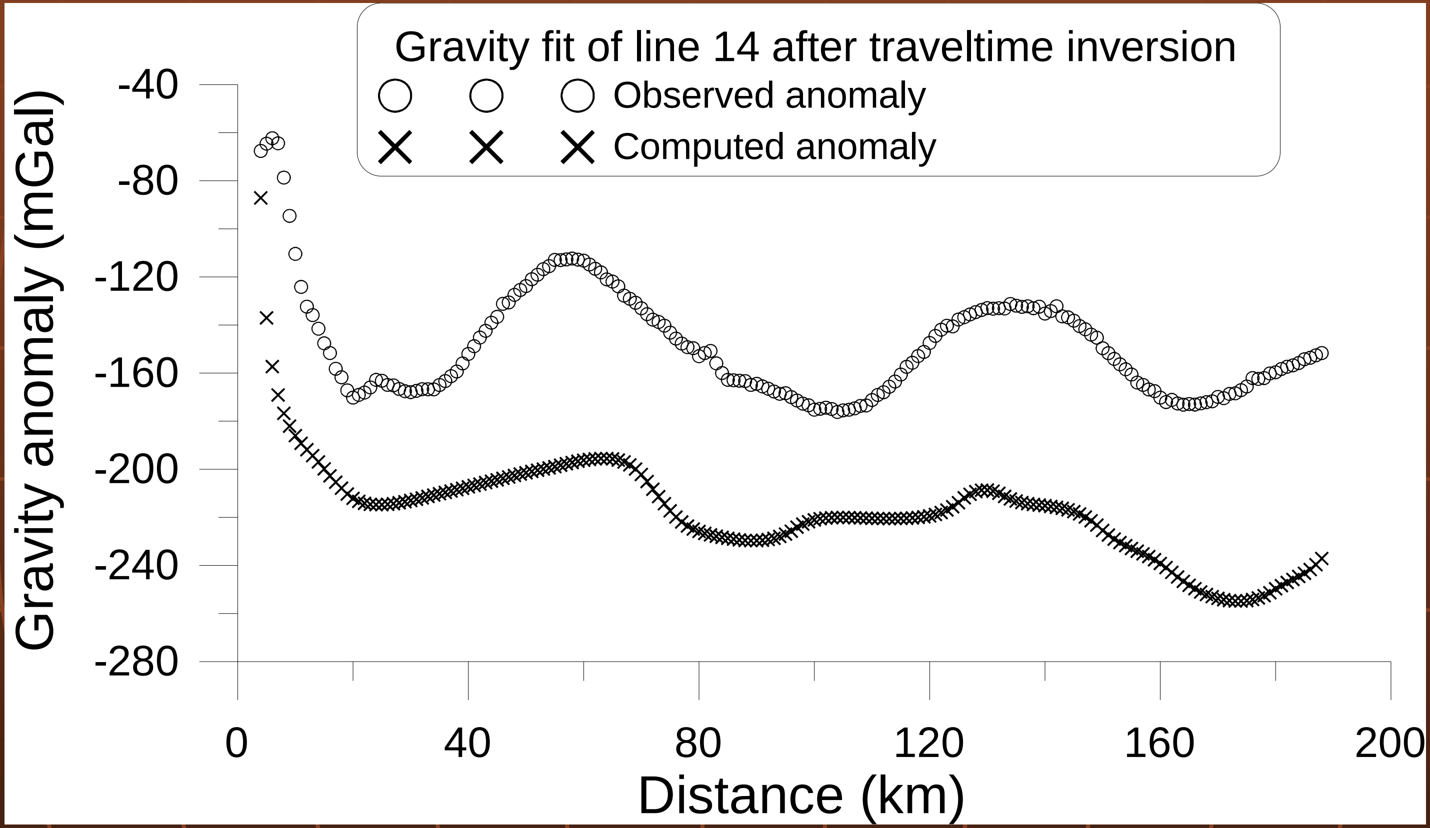
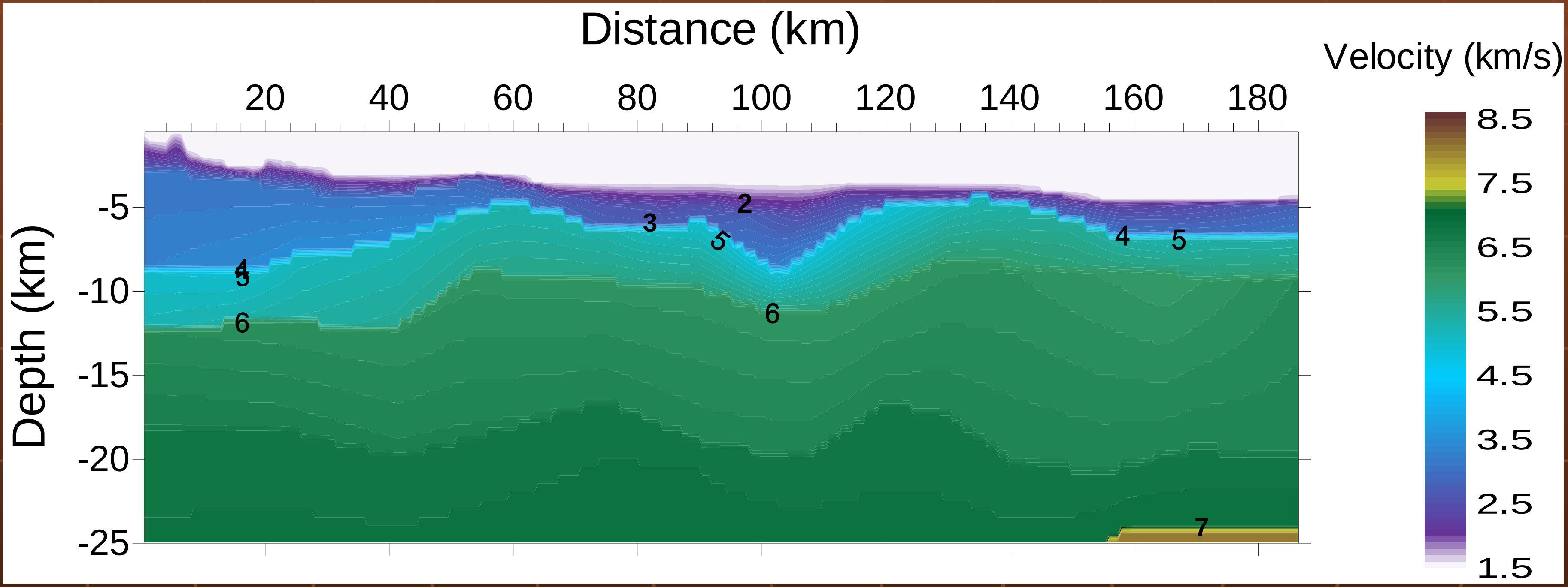

Gravity fit of line 14 after traveltime inversion
Observed anomaly
Computed anomaly
-40
-80
-120
-160
-200
-240
-280
Gravity anomaly (mGal)
0
40
80
120
160
200
Distance (km)

Line 14



Velocity model obtained by linearized tomographic invetsion

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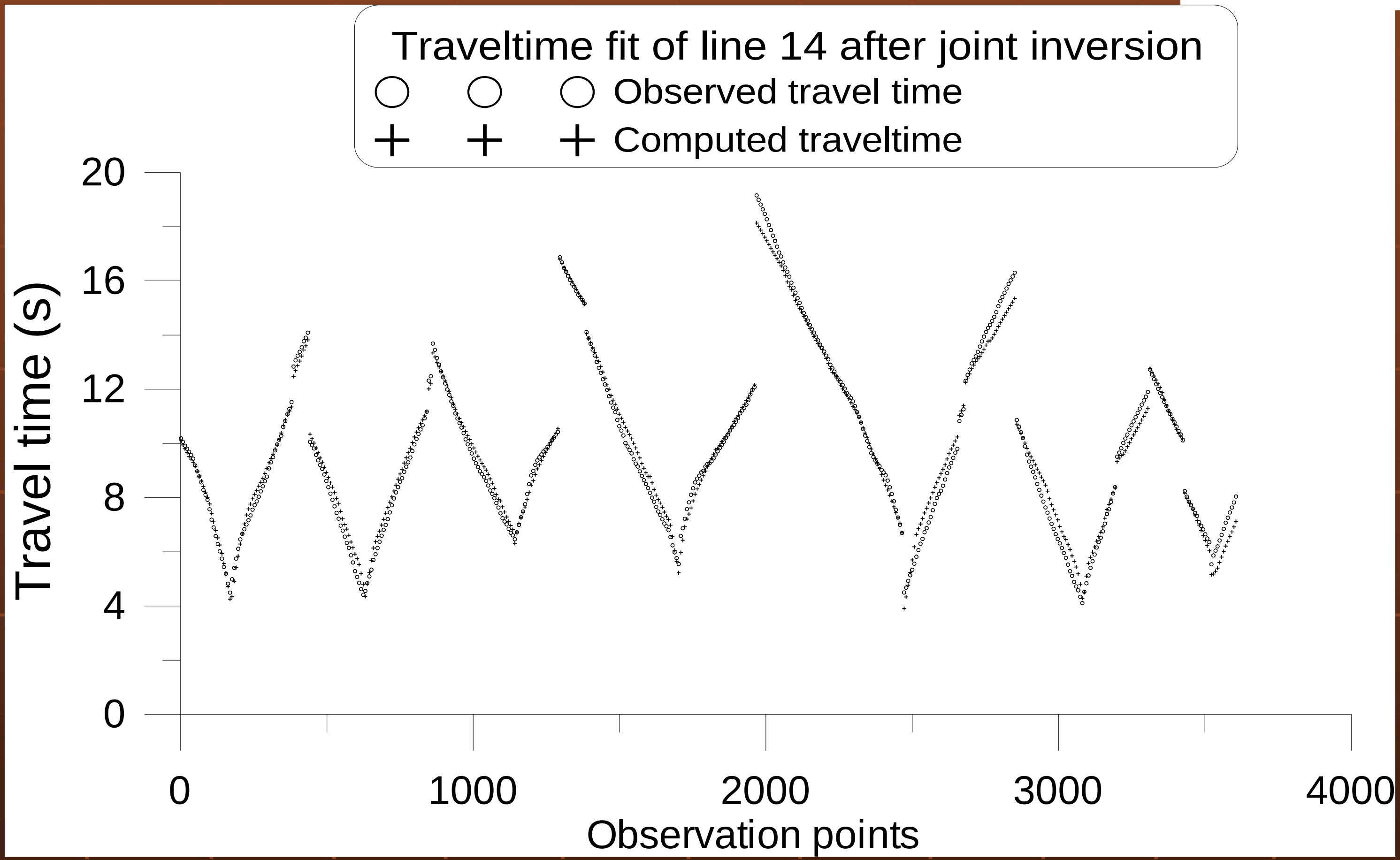

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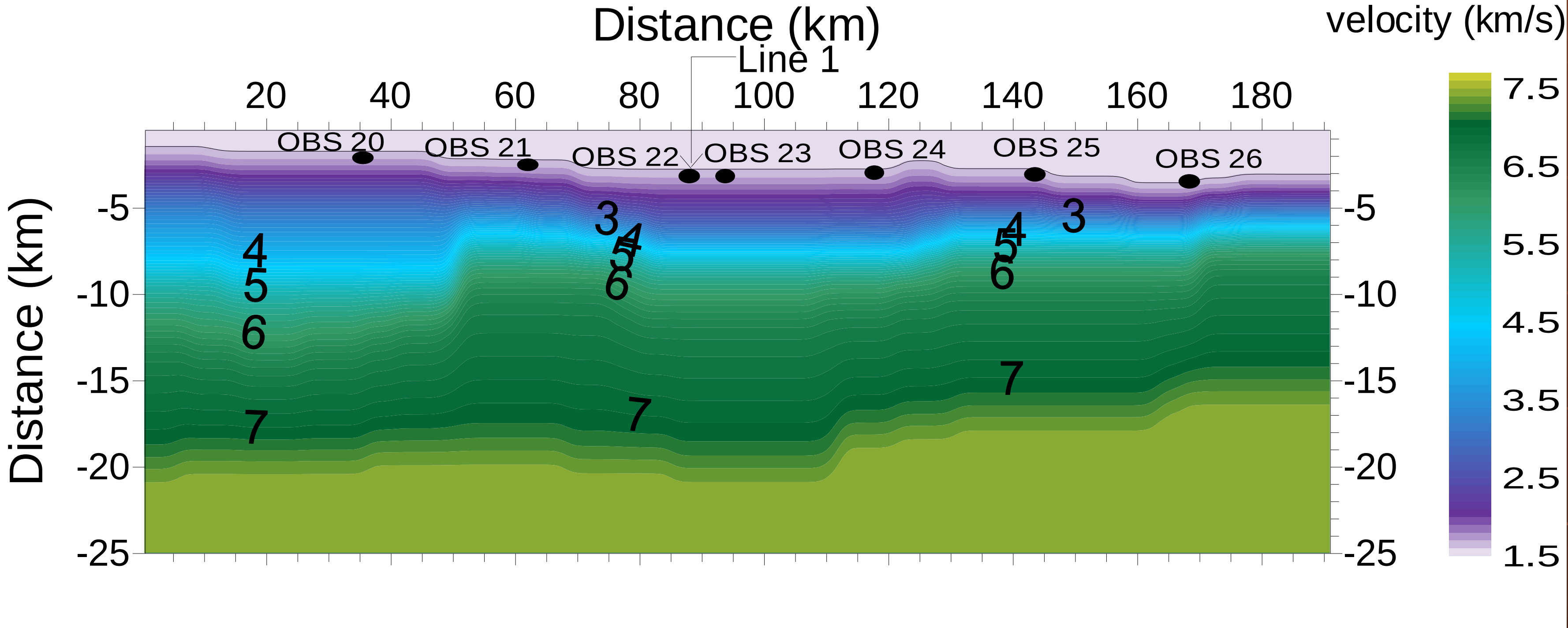
Line 14



Velocity model after joint inversion

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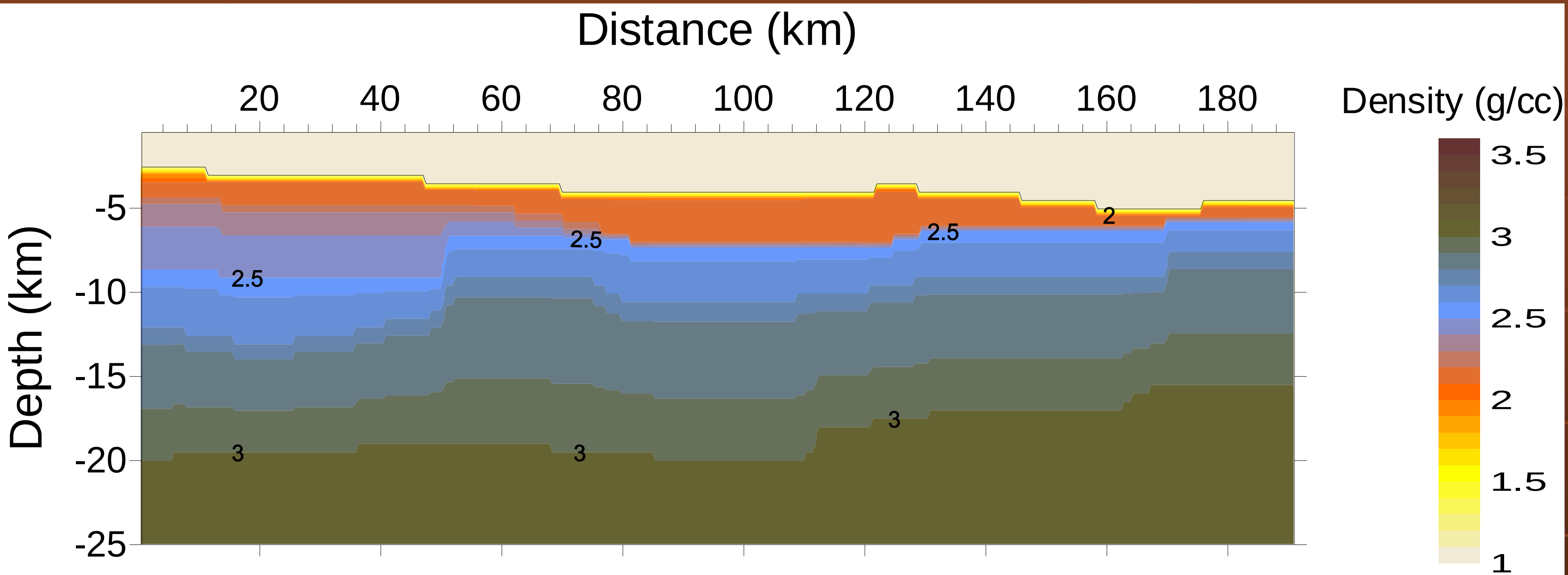
Line 14



Density model obtained after joint inversion

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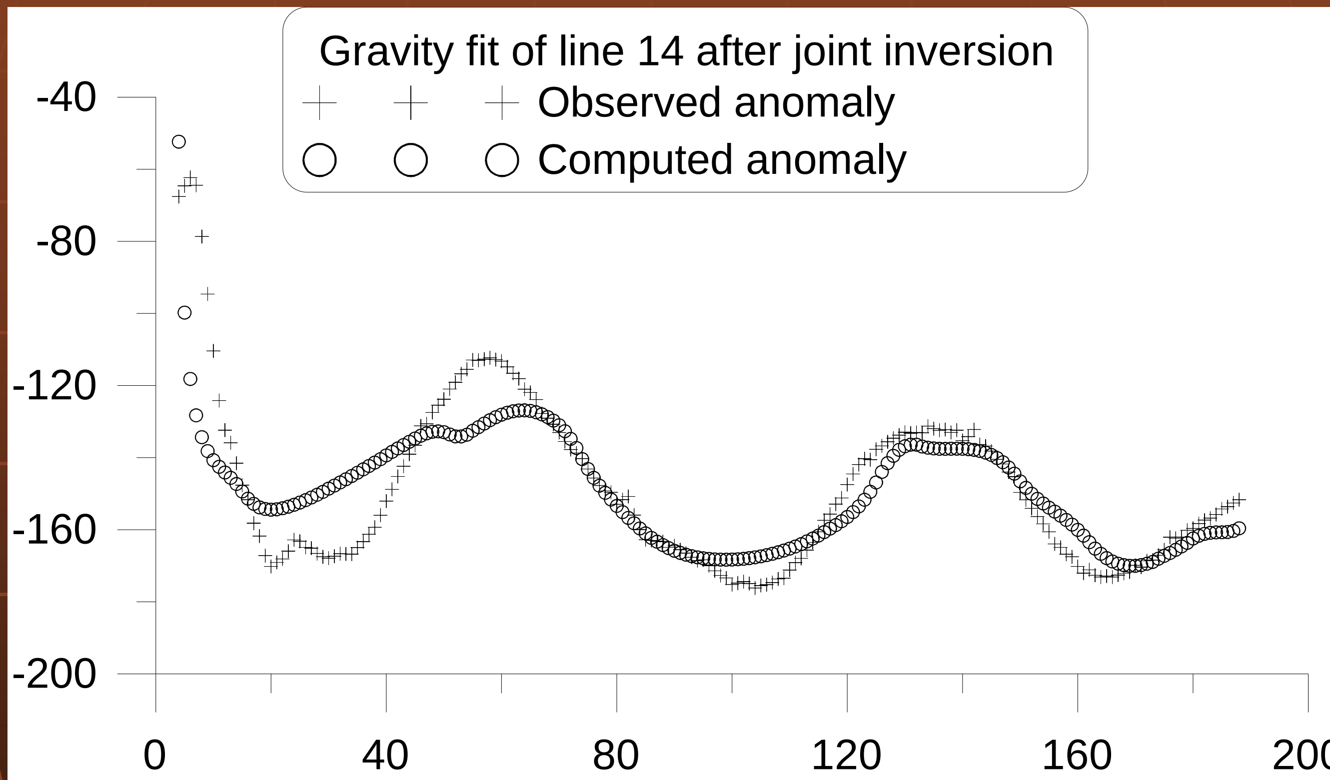

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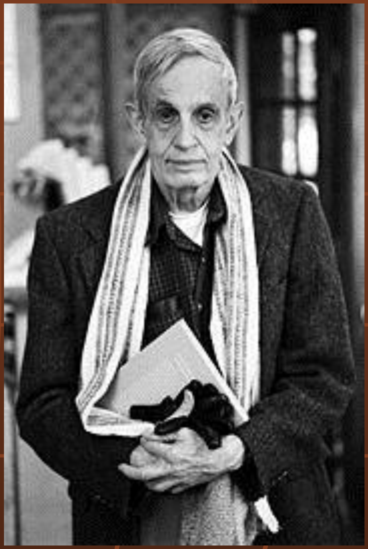
Problem with Global Criterion Method

- Choice of weights → non-trivial!

Story of a Genius Prof. John Forbes Nash, Jr

- Hollywood movie: A beautiful mind → focuses on Nash's mathematical genius and apparent struggle with paranoid schizophrenia
- Undergard: Carnegie Tech;
  PhD: Princeton
- Letter of recommendation
  This man is a genius
  [Carnegie Tech professor, R.J. Duffin]



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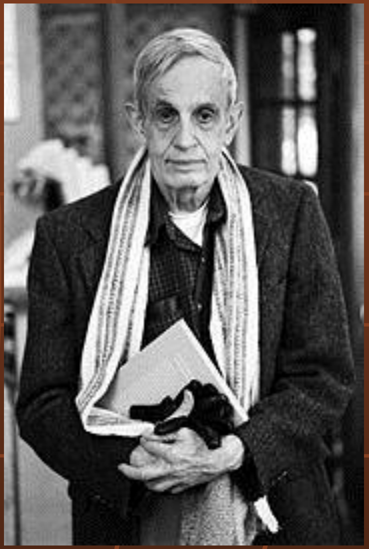
Story of a Genius Prof. John Forbes Nash, Jr

- Earned a PhD in 1950 with a 28 page dissertation on ***game theory***
- Developed the properties and concepts of ***Nash Equilibrium***



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Classical Game Theory

- Basic Definitions

A game consists of:

1) A set of ***Players***

2) A set of ***Strategies***, dictating what action a player can take.

3) A ***pay-off function***, a reward for a given set of strategy choices
eg. Money, happiness

The aim of the game:

Each player wants to optimize their own pay-off.

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Nash Equilibrium

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- A Nash equilibrium is a situation where each player has chosen a strategy which has given them the best possible outcome given what the other players have chosen to.
- If the game was played again then no player could improve by being the only person to change their strategy.

Prisoner's Dilemma

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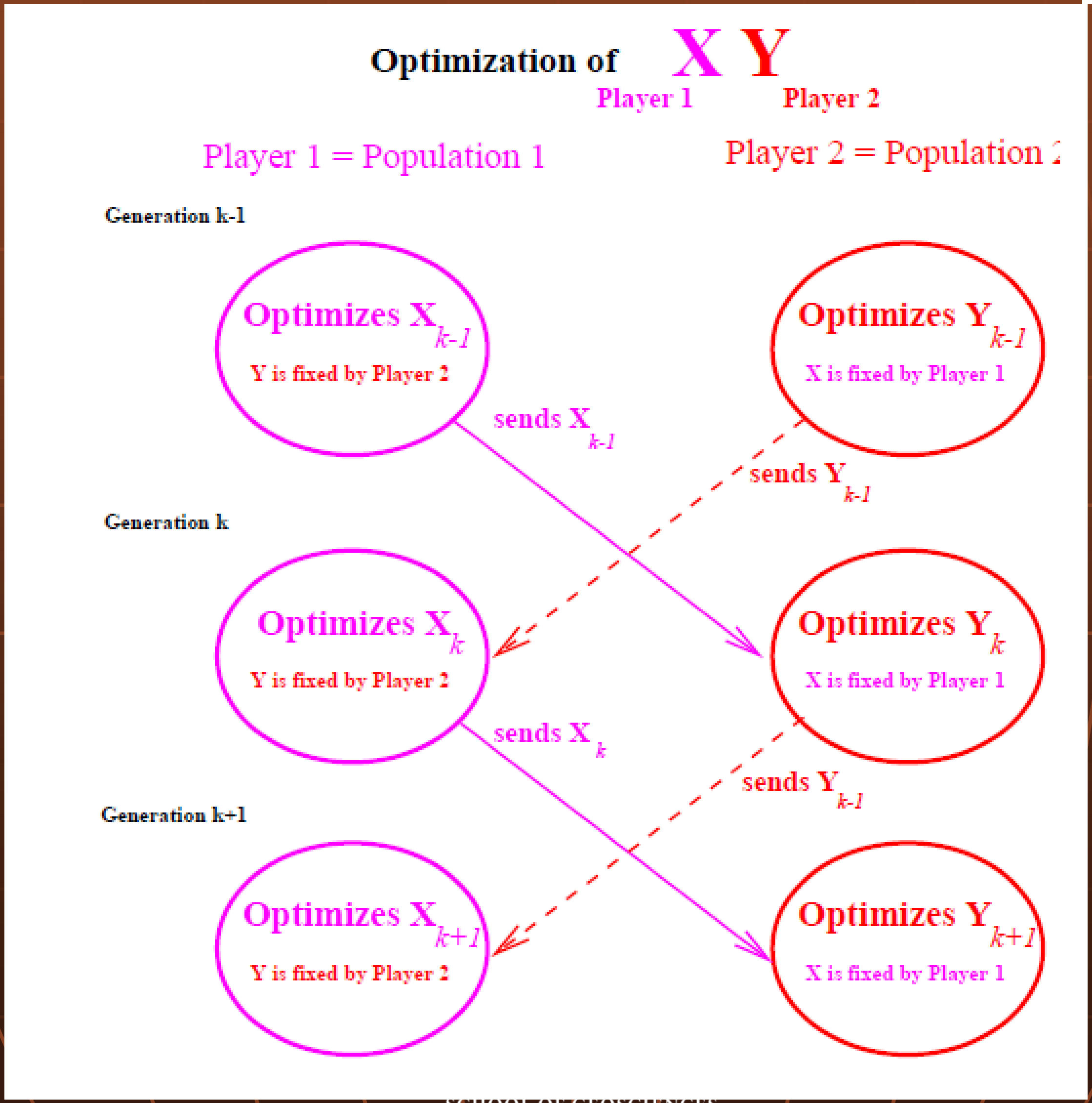
- The **prisoner's dilemma** is a canonical example of a game, that shows why two individuals might not cooperate, even if it appears that it is in their best interest to do so.
- Two men are arrested, but the police do not possess enough information for a conviction.
- Following the separation of the two men, the police offer both a similar deal—
  - if one testifies against his partner (defects/betrays), and the other remains silent (cooperates/assists), the betrayer goes free and the cooperator receives the full one-year sentence. If both remain silent, both are sentenced to only one month in jail for a minor charge. If each 'rats out' the other, each receives a three-month sentence.
  - Each prisoner must choose either to betray or remain silent; the decision of each is kept quiet.

Prisoner's Dilemma

	Prisoner B stays silent (*cooperates*)	Prisoner B confesses (*defects*)
Prisoner A stays silent (*cooperates*)		
Prisoner A confesses (*defects*)		

What should they do?If it is supposed here that each player is only concerned with lessening his time in jail, the game becomes a non-zero sum game where the two players may either assist or betray the other. In the game, the sole worry of the prisoners seems to be increasing his own reward. The interesting symmetry of this problem is that the logical decision leads both to betray the other, even though their individual 'prize' would be greater if they cooperated.

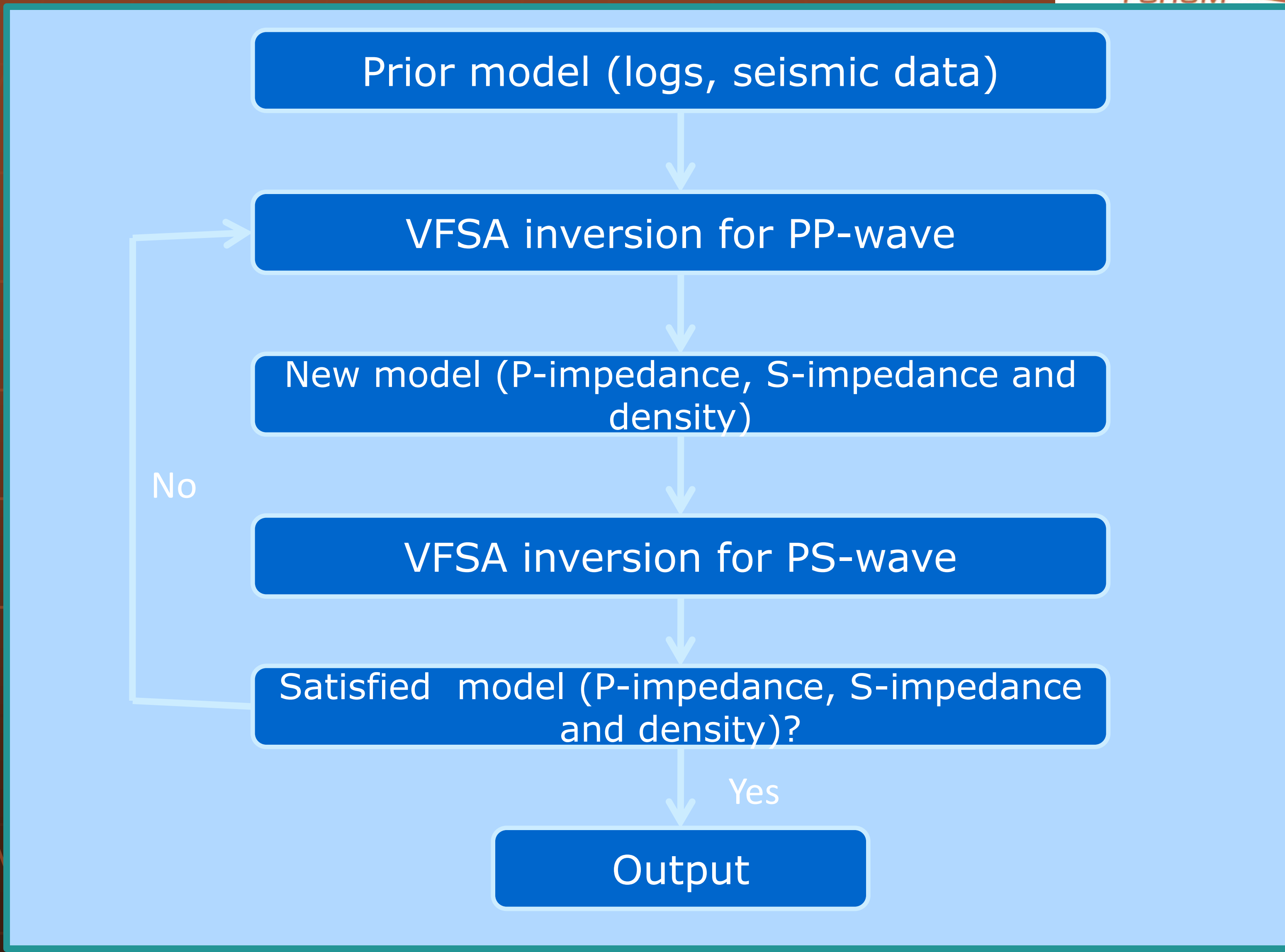
Game Theoretical MOO



Example – Joint inversion of PP and PS data

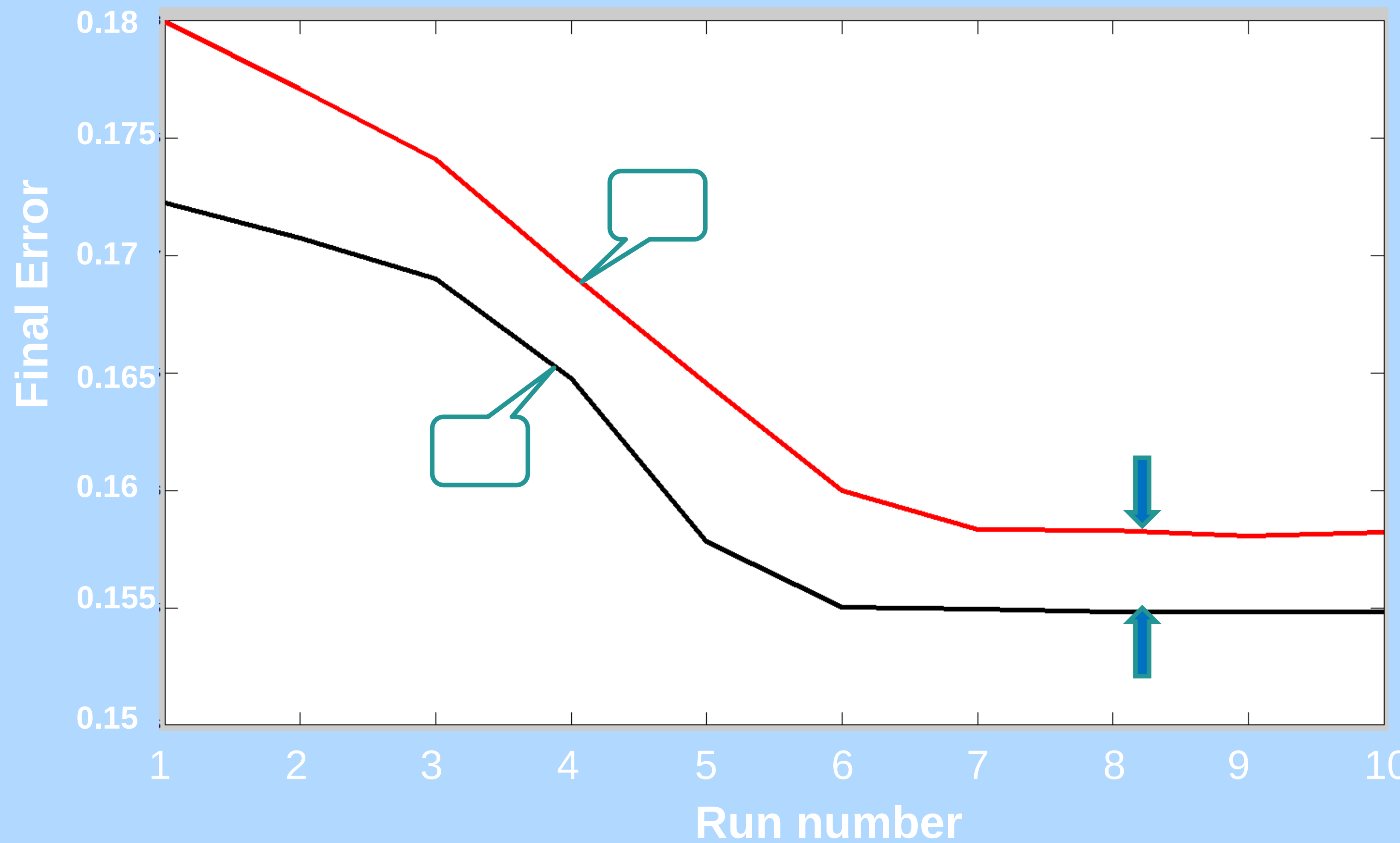
- Weighted Global Criterion Method
- Game Theoretical

GT Inversion workflow



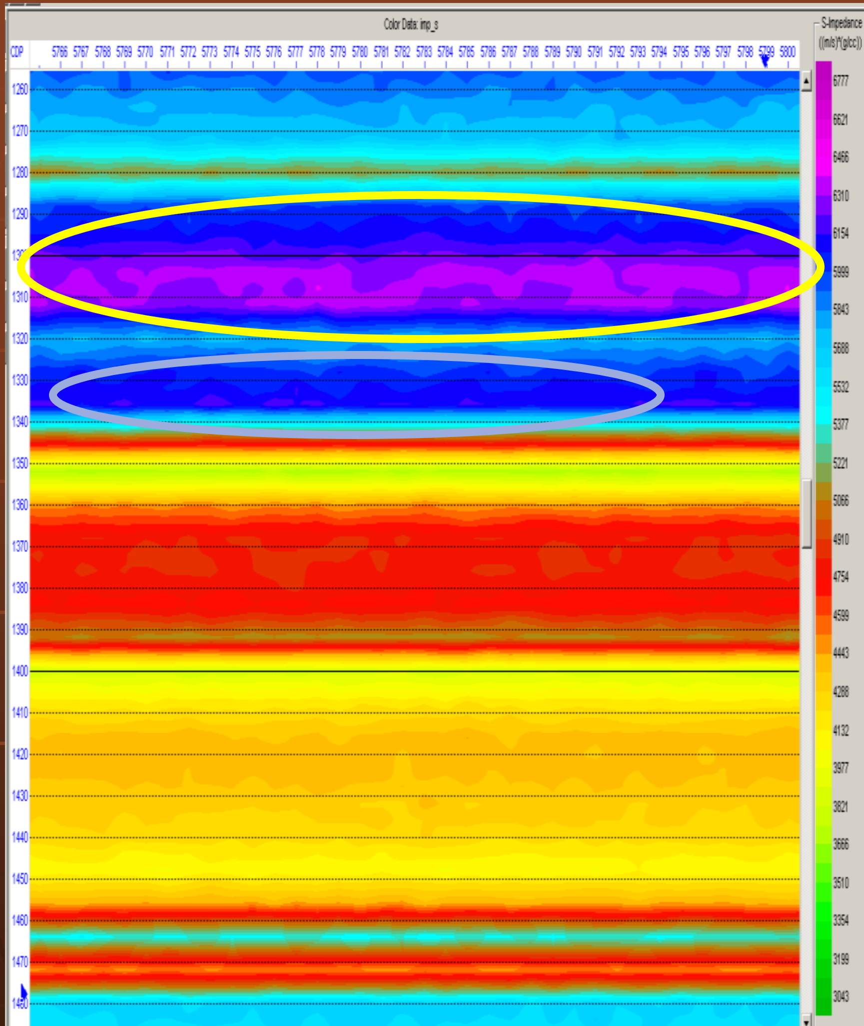
Flow chart of GT inversion

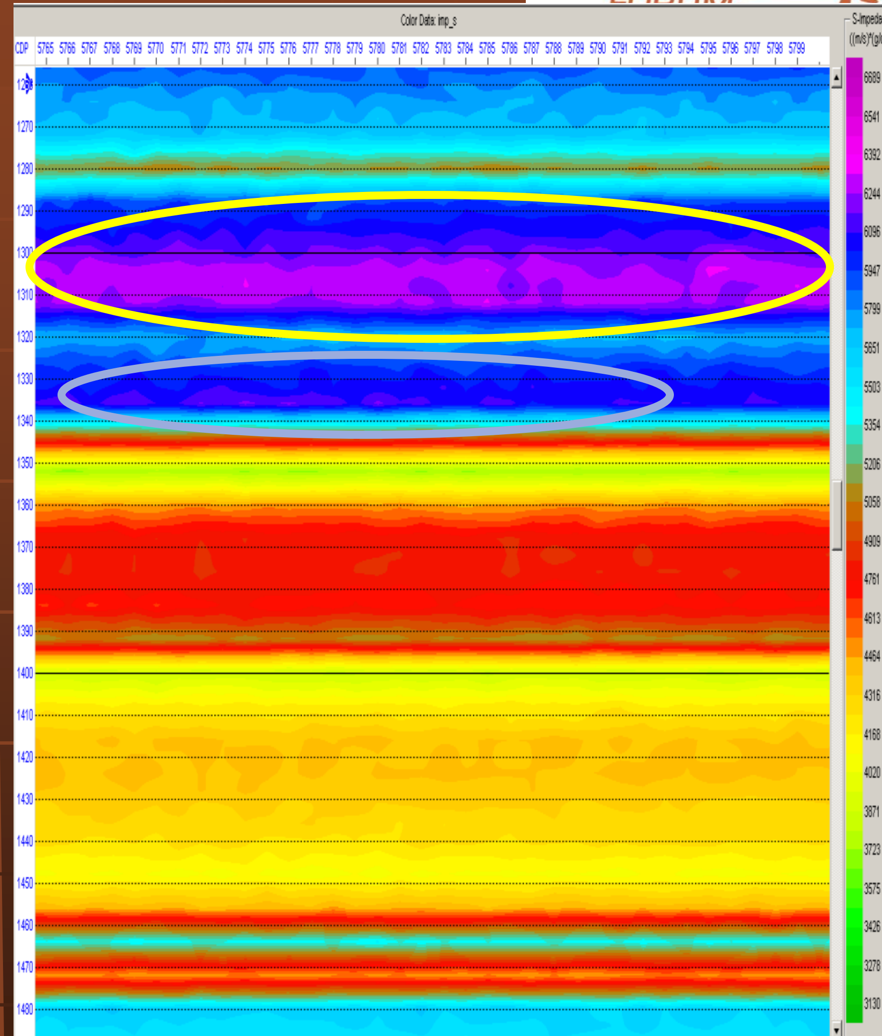
Prestack joint inversion



Cost function error analysis of GT inversion for PP & PS wave

Prestack joint inversion





One step inversion for pp & ps **Two step inversion for pp & ps**

S-impedance

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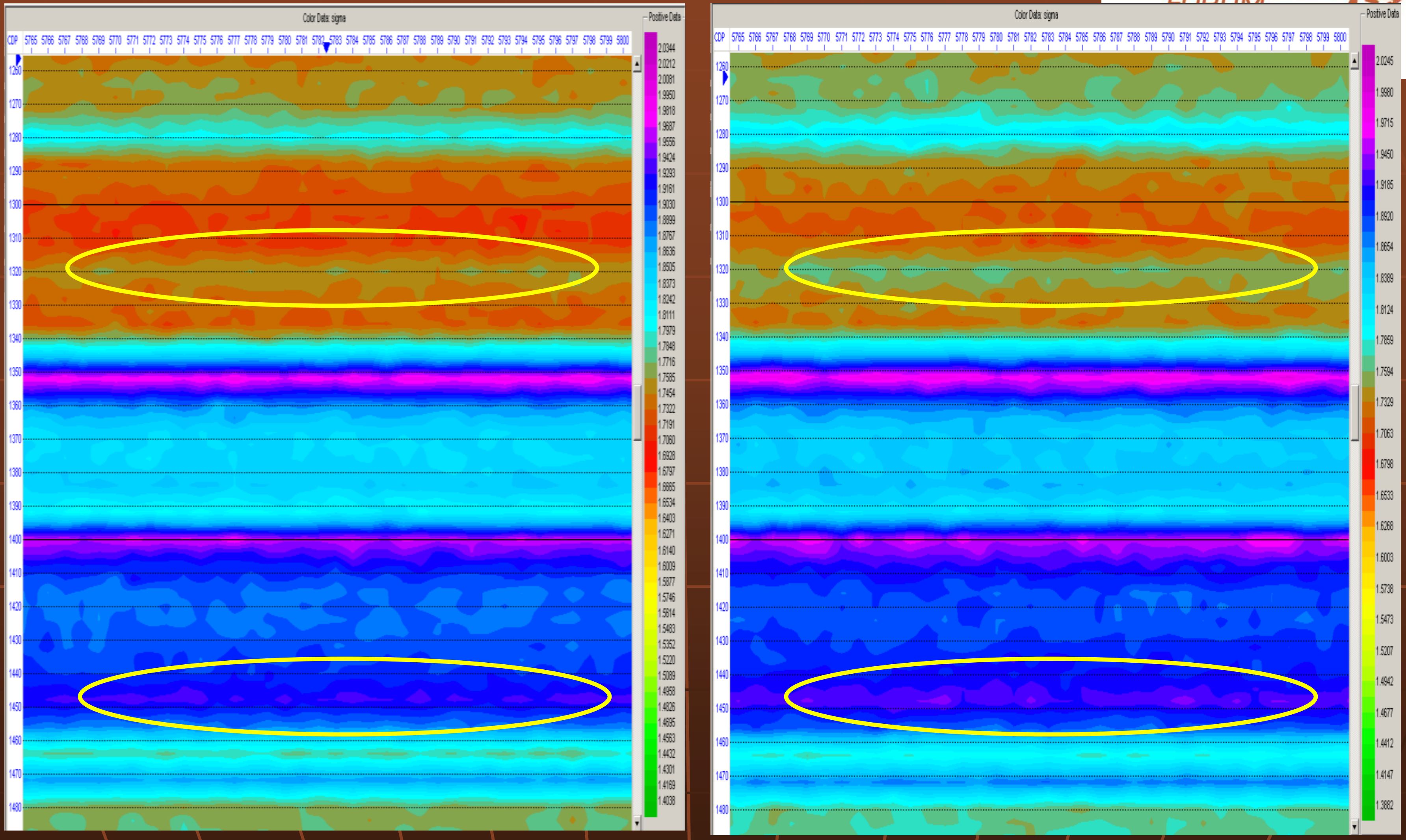
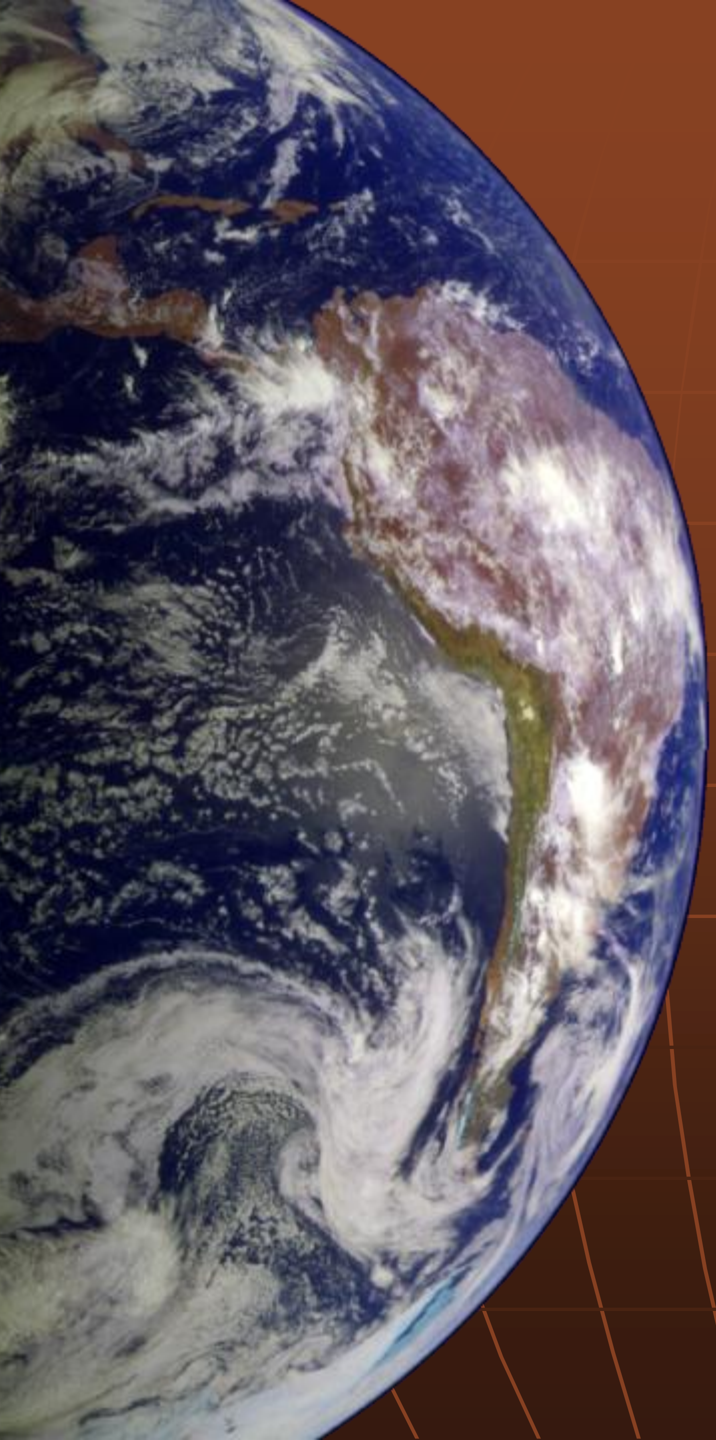
Prestack joint inversion



One step inversion for pp & ps

Two step inversion for pp & ps

Vp/vs



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Where do we go from here?

Uncertainty estimation
Quantum Games

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Quantum Games

The outcome of a quantum system can be probabilistic or even
cannot be described by (local) probability theory.

Quantum

Game Theoretical MOO