Applications for vector coordinate systems of 3-D converted-wave data

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Recently, 3-D four-component marine ocean-bottom cable (OBC) data has come into broad use for a variety of applications. However, understanding how these new components and their associated acquisition and processing coordinate systems are related to the vector wavefield has become a challenging task for the exploration industry. The compressional wavefield (P-wave) is obtained from the pressure response of the hydrophone plus the particle-motion response from the vertical geophone. A converted P-wave to shear wavefield (PS-wave) is obtained from two horizontal geophone components oriented in an acquisition coordinate system of in-line and cross-line particle motion.

The coordinate system familiar in conventional 3-D seismic data describes the position in space of shots, receivers, and image points. This is sufficient when dealing with a single wavefield of seismic data such as the P-wave. However, when 3-D multicomponent data are acquired, whether using multicomponent sources or receivers, an additional coordinate system is needed to specify the vector components of particle motion. This paper focuses on the polarization (not positional) coordinate systems of particle motion for vector wavefields that are acquired by multicomponent 3-D data. P-waves are polarized predominantly in the vertical direction, whereas shear waves (S-waves) are polarized in a horizontal plane having any azimuth.

This paper also focuses on PS-wave exploration applications associated with the different coordinate systems. For example, there are many uses of PS-wave data that have been transformed from acquisition coordinates to source coordinates of radial and transverse S-wave motion. In this case only the radial component oriented in the source-receiver azimuth is retained as an estimate of S-wave reflectivity. One of the main uses is for discrimination of the matrix and fluid types. S-waves yield insights into the nature of subsurface lithologies and the nature of the pore-saturating fluids because they are not affected appreciably by different pore fluids such as gas. The types of information available from converted waves, useful for lithology discrimination, include layer-based $V_p/V_s$ velocity analysis based on correlations or isochron measurements. Another indicator of lithology is horizon reflectivity differences. Since $P$-waves and $S$-waves respond to different rock properties, the strengths of a $P$- or $S$-wave reflection from the same interface may differ greatly. One of the most dramatic applications of converted waves is in structural imaging within gas chimneys and beneath shallow-gas overburdens. The use of converted waves for depth imaging is also becoming common. Independent prestack depth images of the base of salt, using the $P$-wave and PS-wave data, can yield important confirmations as to the validity of a structural interpretation.

A very important use of PS-waves is for vertical fracture detection, accomplished by further transforming common azimuth prestack data to radial and transverse receiver coordinates. This provides the necessary data for four-component rotations and layer-stripping algorithms to analyze $S$-waves polarized into slow and fast arrivals (a form of anisotropy known as birefringence). One application is to enhance the fast and slow $S$-wave for lithology discrimination, i.e., calculation of $V_p/V_s$ and inverting PS-waves for impedance. Fracture intensity and fracture direction (related to the degree and direction of anisotropy) may be quantified between specified horizons of interest to define preferred zones of fracturing that can have a major impact on reservoir models. Models that incorporate fracture density and orientation can lead to optimal recovery of hydrocarbons and more refined use of 4-D time-lapse measurements.
**Acquisition coordinates.** Multicomponent data are generally acquired with a rectangular polarization coordinate system for either 2-D or 3-D surveys. That is, at any one receiver location there are three-component seismic traces defined as \( s_1(t) \), \( s_2(t) \), and \( s_3(t) \) or in terms of \( x \), \( y \), and \( z \) as \( s_x(t) \), \( s_y(t) \), and \( s_z(t) \), where \( t \) is recording time. These are typically called the in-line, cross-line, and vertical components of motion for the corresponding directions, respectively, where the in-line is parallel to some reference frame such as a receiver line (Figure 1) or a shot line in the case of multicomponent sources.

This system is generally a right-handed system as shown in Figure 1. The in-line component points in the positive receiver line direction, the cross-line points at 90° to the right, and the vertical points downward. It is right-handed in the sense that the \( y \) component is clockwise from the \( x \) component when looking in the positive \( z \) direction. Tapping in the direction of the arrows (positive direction) yields positive outputs in agreement with SEG polarity standards.

**Land.** In most cases, the orientation of multicomponents can be carefully positioned for land surveys. For example, three-component receivers can have leveling bubbles and be oriented horizontally with compasses to align the in-line in a specified direction. All receivers are oriented the same as illustrated in Figure 1. In the case of \( S \)-wave source data, horizontal vibrators can be operated in the desired orientation at each source point in a similar manner.

**Marine.** For marine multicomponent surveys it is more difficult to accurately position and orient receivers. This is true for all acquisition methodologies to some degree, whether cables are dragged or draped, or for units planted on the seafloor by remotely operated vehicles.

Typically, four components are recorded in marine surveys: The two horizontals, in-line and cross-line, provide \( P \)-wave data. The vertical and hydrophone provide \( P \)-wave data for attenuating water-column reverberations. Figure 2 illustrates an exaggerated situation for an OBC survey in which the receivers are oriented in different directions at each receiver location. Receivers are typically gimballed, however, so the three components remain horizontal and vertical, but the in-line component can point in arbitrary directions. Cables draped on the seafloor will exhibit more variation in orientation than cables that are dragged into place under tension.

Because of the difficulty in planting receivers accurately in the marine environment, the location and orientation must be determined by some independent means. Traveltime triangulation, similar to earthquake location techniques, can locate receivers to within a radius of several percent of the water depth. Arrival times can be obtained from either the actual seismic data or from transducers mounted on the OBC.

To determine the orientation of the horizontal components, first-break polarization analyses rely on direct \( P \)-wave arrivals, as well as \( P \)-wave refractions, \( P \)-wave and \( PS \)-wave reflections that are linearly polarized seismic events. Linear polarization is a displacement property in which the particle motion of a seismic wave oscillates in one direction and traces out a line. By contrast, surface waves on land or water-bottom interface waves are elliptically polarized; parti-
cle motion tends to oscillate in two directions, vertical and horizontal, to trace out an ellipse.

Figure 3 illustrates the essence of linear polarization for a direct arrival recorded on the two horizontal components. Both $s_x$ and $s_y$ have equal amplitude, representing the special case of a bisecting ray for the highlighted source and the orientation of the circled horizontal components in Figure 2. If the samples of the two components are plotted in a hodogram, the response traces out a line. The two components can be rotated mathematically in the computer to simulate preferred directions of polarization, as if the data had been acquired with some other receiver orientation. A $2 \times 2$ rotation operator is applied at each time sample of the $s_x$ and $s_y$ seismograms to yield new seismic traces $s'_x$ and $s'_y$ that represent the seismic wavefield polarization for the new orientation. As an example, applying the desired orientation $\theta$ to the data in Figure 3 will place all the energy on the $s'_x$ component and leave the $s'_y$ devoid of energy. The operator is unitary and does not change the total magnitude of reflections.

Given the two components, $s_x$ and $s_y$, it is a simple matter to estimate (from the direct-arrival energy when the orientation is unknown. This will provide the orientation of the horizontal components as they came to rest on the seafloor. Determining $\theta$ can be done by least-squares methods to find the optimal orientation that minimizes the energy on the $s'_y$ component. An attractive feature of this method is that it can be performed on numerous shots for the same receiver to provide a statistical measure of the orientation. Since an energy criteria, $S'_y(\theta)$, is used, polarity ambiguities of $\pm 180^\circ$ can be resolved by correlating the $s'_x$ component with the hydrophone direct-arrival signal.

In the acquisition coordinate system, the vertical components provide consistent $P$-wave data from station to station. Reflections from a positive impedance contrast result in negative outputs regardless of the horizontal direction of travel. However, $PS$-wave data acquired with this geometry will not have consistent amplitudes and polarization directions from one receiver to the next.

The polarization inconsistencies resulting from the geometry in Figure 2 are not a problem as long as the locations and orientations of the horizontal components are accurately determined. Once the orientations are
known, horizontal components can be rotated mathematically in the computer to simulate preferred directions of polarization, as if the data had been acquired with some other orientation. A simple two-component rotation is performed in which the rotation angle is given by the difference \( \theta = \psi - \phi \), where \( \psi \) is the desired orientation (direction or azimuth) and \( \phi \) is the original or current orientation. If the desired orientation (is the receiver line direction and rotations are applied to the horizontal components in Figure 2, the coordinate system of Figure 1 can be achieved.

Even this system, however, will not provide consistent PS-wave data from one receiver station to the next. Sources from different directions with respect to a receiver will result in varying amounts of energy on the in-line and cross-line components. This is illustrated in Figure 4. It shows a common-receiver gather of the in-line and cross-line components in the circle for the shot line passing near the receiver in Figure 2. Each trace is associated with a different shot. The amplitude on the in-line goes through zero when the shot is close and perpendicular to the receiver. Conversely, the amplitude on the cross-line becomes a maximum for these shots. Accounting for source-related azimuthal variations in amplitude is the subject of the next section.

**Source coordinates.** To obtain consistency of PS-waves from one receiver station to the next, horizontal components are rotated into a source-centered cylindrical coordinate system. This is accomplished by setting \( \psi \) to the direction pointing away from the source (Figure 5) so that each three-component receiver acquires a different orientation for each shot, defined by the source-to-receiver azimuth. The result is similar to that shown in the hodogram in Figure 3. After rotation, \( s'_r = s_r \) and becomes the radial component, and \( s'_t = s_t \) to become the transverse component. Now all the \( s_r \) components point away from the source in the positive radial direction and all the \( s_t \) components point in the clockwise direction around the shot. There will be a different set of rotations and source-centered coordinate system for each shot.

The particle motion of PS-waves oscillates in the direction of P-wave propagation (in an isotropic medium) when it converts to an S-wave at an interface. For flat-layer geology this direction is the source-to-receiver direction. It is important to note that the angle \( \psi \) depends only on the positions of the source and receiver so that the radial component is aligned with PS-wave particle motion. No attempt is made to optimize particle-motion energy by examining the polarization of all the energy on the radial component and pointing the \( s_r \) component in those directions. Only the direct arrival is used, as discussed above, to determine horizontal component orientations.

Another important property of PS-waves is that reflections have different amplitude-variation-with-offset (AVO) response than P-waves. At near offsets, reflection amplitudes are proportional to the sine of the angle of incidence and change polarity at zero offset (Figure 4). To remove this inconsistency for 2-D data, the source coordinate rotation effectively reverses the polarity of the in-line component for negative offsets. Receivers on either side of the source have a source-receiver azimuth 180° different. This is precisely the required result, and after rotation, positive and negative offset radial-component reflections have the same polarity, as shown in Figure 6.

This is also true for 3-D data, since each azimuthal direction away from a shot (positive radial direction) can be considered as positive offsets from a 2-D line. In this regard, the source-centered coordinate system is in agreement with the SEG polarity standard. PS-wave reflections arising from a positive P-wave impulse off a positive S-wave impedance contrast will have a negative output for all propagation directions.

Note in Figure 6 that arrivals appear on the transverse component at about 400 ms. These can be energy arriving out of plane that must be handled properly by migration or they could be due to S-wave birefringence or perhaps both. This will be discussed in the next section.

There are many uses of PS-wave data that have been transformed from the acquisition coordinates to source coordinates of radial and transverse shear-wave motion. In these cases only the radial component, \( s_r \), oriented in the source-receiver azimuth is retained as an estimate of shear-wave reflectivity. Assumptions of isotropy and flat-layer geology are made at this
stage in the processing (similar to conventional $P$-wave processing)—implying that all the $PS$-wave energy of interest is on the radial component.

One of the main uses for these data is discrimination of the matrix and fluid types. $S$-waves yield insights into the nature of subsurface lithologies and the nature of the pore-saturating fluids because they are not affected appreciably by different pore fluids.

Lithology discrimination. The types of information available from radial component $PS$-waves, useful for lithology discrimination, include layer-based $V_p/V_s$ velocity analysis based on correlations between $P$-wave and $PS$-wave reflectivity. $P$-wave traces are correlated with a suit of transformed $PS$-wave traces to establish optimal $V_p/V_s$ that defines which reflections belong to the same interface in the stratigraphic column. Transformation consists of compressing and shifting $PS$-wave traces based on average and interval $V_p/V_s$. Correlation peaks as a function of $P$-wave time, and $V_p/V_s$ can be picked in a similar manner as semblance peaks for determining stacking velocities.

Figure 10. Multicomponent example from the East Cameron area in the Gulf of Mexico. $PS$-waves are not as sensitive to the presence of gas.

Figure 11. Prestack depth images of the Mahogany salt body in the GOM for $P$-waves and $PS$-waves. Possible subsalt events are present in both images.

Figure 12. Strong $S$-wave birefringence ($S$-wave splitting) from the Oseberg 3-D survey in the North Sea. Radial and transverse components are prestack azimuth gathers in 10° azimuth bins. The radial component is fairly consistent with azimuth but the transverse exhibits significant interference that results in polarity changes every 90°.
Traveltimes of P-waves and S-waves arise, particularly for applications related to lithology discrimination. The next section discusses prestack azimuth processing within a source and receiver coordinate system to unravel the effects of splitting for inversion and fracture detection.

Isochron measurements (as long as the P- and S-wave reflections are from the same horizons) for mapping lateral variations in lithology. Isochron VP/VS ratios have been used to indicate where dolomitized reservoir facies exist in the Scipio trend in southern Michigan.

In addition to traveltimes, another indicator of lithology is horizon reflectivity differences. Because P-waves and S-waves respond to different rock properties, the strengths of a P- or S-wave reflection from the same interface may differ greatly. The North Sea 3-D Oseberg data set provides a good example in which P-wave and S-wave amplitudes can help delineate reservoir sands and channels. Figure 8 shows representative stacks from the 3-D volume that are used to compute P-wave and S-wave impedances. The reservoir is just below the P-wave reflection at 2.2 seconds and below the PS-wave reflection at 4.1 seconds. A map of VP/VS impedance (P-wave divided by S-wave), projected onto the target horizon, is shown in Figure 9. Associated with the reservoir sand body is a distinctive low VP/VS impedance signature.

Imaging through gas. One of the most dramatic applications of converted waves is in structural imaging in the presence of gas chimneys and beneath shallow-gas overburdens. In the North Sea, shallow gas channels often distort P-wave images of potential reservoirs by introducing delays and severe attenuation. Also, gas in sediments above reservoirs can render P-wave images useless for mapping detailed trapping mechanisms. PS-waves, however, are nearly unaffected by the gas and can yield excellent images in these cases.

Gas can also be a problem in the Gulf of Mexico. Figure 10 shows an example from the East Cameron area. Although the P-wave data acquired in OBC surveys can be an improvement over conventional streamer data, the influences of gas in the form of traveltimes, sags, reverberations, and attenuation are still present. The PS-wave data, however, can provide a clearer image through these zones.

Depth imaging. The use of converted waves for depth imaging is also beginning to draw considerable interest. Independent prestack depth images of the base of salt or basalt, using P-wave and PS-wave data, can yield important confirmations as to the validity of a structural interpretation. Figure 11 shows preliminary prestack-depth images of the Mahogany salt body in the GOM for P-waves and PS-waves. An initial P-wave velocity-depth model has been tailored using estimates of average VP/VS to drive the PS-wave prestack depth migration. Although possible subsalt events are present in both images, further refinement of the P-wave and S-wave velocities will enhance subsalt structures.

The radial-source coordinate system provides a very robust PS-wave field for the applications described above even in the presence of weak birefringence or S-wave splitting. However, when splitting is significant, limitations and complications in the S-waves arise, particularly for applications related to lithology discrimination. The next section discusses prestack azimuth processing within a source and receiver coordinate system to unravel the effects of splitting for inversion and fracture detection.
Source and receiver coordinates. S-waves tend to travel in pairs in azimuthally anisotropic media and the two are polarized perpendicular to each other. Azimuthal anisotropy is where velocity changes with azimuthal direction, as opposed to position. Only when the medium is perfectly isotropic, where velocity is the same for waves traveling in any direction, do the two S-waves have the same velocity. If the velocity of the two differs slightly they will separate as they propagate; the fast S-wave will travel in advance of the slow one, which lags behind. Over large distances they will arrive at a receiver at slightly different times. Typical differences in S-wave velocity range from several percent to about 10 percent.

The manifestation of S-wave birefringence can be seen in Figure 6 on the transverse component. In this flat-layer medium there is very little energy for the first 500 ms, after which PS-wave reflections immerge but are weaker than on the radial component. As discussed above, the direct arrival is a P-wave and is polarized in the radial direction. The earliest PS-wave reflections are also polarized in the radial direction and represent S-waves that have not separated from one another; they appear to be polarized in the source-receiver radial direction (the direction in which they were created). Later PS-wave reflections, however, represent two S-waves that have separated enough (a portion of the dominant wavelength) to interfere with each other and to be detected on the transverse component. S-wave splitting is cumulative and increases with recorded time, as shown in Figure 6. When birefringence is weaker, the appearance of splitting occurs later in the record.

The interference of split S-waves is quite complicated and varies with propagation azimuth. Figure 12 shows an example of strong S-wave birefringence from the Oseberg 3-D survey discussed above. It represents a prestack azimuth supergather from a single location in the survey and is characteristic of the interference of fast and slow S-waves in birefringent media. Radial and transverse components are gathered into 10° azimuth bins and stacked over offset after applying normal moveout with a single velocity function. The radial component is fairly constant with azimuth, but the transverse component exhibits significant amplitude variations and a polarity reversal every 90°. At the polarity reversal amplitude of the transverse component passes through zero, and these azimuths correspond to the principal symmetry axes of the anisotropic medium in which S-wave splitting does not occur. These are the directions of the fast and slow S-waves.

The objective of prestack azimuth processing is to combine all the S-wave energy into the fast and slow S-waves. Simply stacking all the radial components will result in only an estimate of the average radial component; an average of the S-wave interference. Stacking all the transverse components will result in a canceling effect and relatively little reflected energy, a response that could be erroneously interpreted as resulting from an isotropic medium. There is no energy on the transverse component in an isotropic flat-layered medium.

To unravel the polarization effects of S-wave birefringence a combined source-centered and receiver-centered coordinate system is utilized and a 1-D model of S-wave splitting is assumed. This polarization coordinate system has two indices for each trace, sij, where the first index refers to sources and the second index refers to receivers. In this system, there is a radial, r, and transverse, t, source and receiver.

As an example, Figure 13 shows the geometry for a single image point of this coordinate system, where a selected azimuth is defined as the radial direction and 90° to that is the transverse direction (the vertical component is not shown). For the two directions a number of shot-receiver pairs from different offsets are shown, all of which have the same P-wave to S-wave conversion point (image point). Note that both the source and receiver coordinates are aligned with...
each other and that this coordinate system now has four components. There are radial sources detected by radial and transverse receivers, \(s_{rr}\) and \(s_{rt}\), and there are also transverse sources detected by radial and transverse receivers, \(s_{rt}\) and \(s_{tt}\). Other radial and transverse directions can contribute to the total output at a single image point.

A four-component rotation operator (Alford rotation) is used to simultaneously orient these data into the principal symmetry axis direction in which the new components \(s'_{rr}\) and \(s'_{rt}\) will be minimized. In this orientation the components \(s'_{rr}\) and \(s'_{tt}\) correspond to the fast and slow \(S\)-wave, \(S_1\) and \(S_2\) respectively.

An additional step is necessary to unravel the polarization interference effects of \(S\)-wave birefringence, i.e., aligning the \(S_1\) and \(S_2\) after the off-diagonal terms in the Alford rotation have been minimized. Correlating the two \(S\)-waves after rotation and shifting the receiver components of \(S_1\) to match \(S_1\) removes the delay between the fast and slow \(S\)-waves. It is important to emphasize that this procedure be applied in a layer-stripping manner (shallow to deep) because the time separation between \(S_1\) and \(S_2\) is cumulative with recorded time and the orientation of the principal symmetry axes may change with depth.

Regrouping the radial and transverse data from Figure 12 results in the four \(S\)-wave components for Alford rotation and layer stripping (Figure 14). Five seconds of data are shown. The two components on the left are the same as those in Figure 12 and correspond to \(s_{rr}\) and \(s_{rt}\), the radial and transverse receiver components for the radial-source direction. The two components on the right are \(s_{rt}\) and \(s_{tt}\), the transverse and radial-receiver components for the transverse-source direction. The latter components are the same traces as in the left two components, except shifted in azimuth by 90°, and \(s_{rt}\) is reversed polarity. The polarity reversal can be seen by examining Figure 13 in which receivers for the transverse source direction are equivalent to radial and transverse receivers in the source-centered coordinate system, i.e., \(s_{rt} = s_{rr}\) and \(s_{rt} = -s_{tt}\).

Figure 15 shows the results after applying the Alford rotation and layer stripping to the data in Figure 14 in two layers, first in an upper layer from 0.3 to 2.5 seconds and then from 2.5 to 5.0 seconds. A majority of the interference effects has been combined from the original four components into the fast and slow \(S\)-waves for each of the azimuths. Note the significant difference between the fast and slow \(S\)-wave reflectivity, particularly in the first two seconds of data.

**Inversion of \(P\) and \(PS\)-waves for impedance.** There are many uses of the enhanced \(S\)-wave components \(S_1\) and \(S_2\) after prestack azimuth processing. One important application is lithologic discrimination, as discussed above. Enhanced \(S\)-waves will provide more accurate \(V_p/V_s\) estimates when reflec-
Fracture detection. A very important use of Alford rotation and layer stripping is for reservoir fracture detection. Fracture intensity and fracture direction (related to the degree and direction of anisotropy) may be quantified between specified horizons of interest to define preferred zones of fracturing that can have a major impact on reservoir models. The time separation of $S_1$ and $S_2$ is directly related to fracture intensity, i.e., fracture density is proportional to the degree of azimuthal anisotropy. Another indicator of fracture intensity is the impedance contrast between $S_1$ and $S_2$. As fracture intensity approaches zero, the fast and slow $S$-waves become more similar. Fracture direction is given by the orientation of the fast $S$-wave, $S_T$ polarization.

Figure 16 shows east-west (radial-source direction) stacks of the radial and transverse receiver components from a 3-D survey acquired in the Madden Field in Wyoming. Using these two components as well as the transverse-source components from the north-south direction, the orientation of fast $S$-wave and the traveltime difference between $S_1$ and $S_2$ are determined for the purpose of delineating fractures. Proceeding from the top, Figure 17 shows the $S$-wave time difference map (as percent anisotropy) for the shallow reflection sequence between 1.2 and 2.2 seconds. In this case there is little $S$-wave splitting, indicated by the presence of energy only on the radial component. The transverse component does not have a reflection between 1.2 and 2.2 seconds in Figure 16.

After removing the birefringence effects of the overburden, the fracture orientation and density are determined for deeper intervals. The plots in Figure 18 are the fast $S$-wave direction and percent anisotropy, corresponding to possible fracture orientation and fracture density of the target reflections between 2.2 and 3.3 seconds, respectively. Regions of high-percent anisotropy (9% or more) correlate well with the known east-west trending faults superimposed on the maps.

As in many situations, there is an issue of resolution. These fracture property estimates are on average over 1.1 seconds of data, an interval clearly larger than the reservoirs of interest. Fracture detection at finer intervals can be achieved by careful survey design to provide optimal fold, offset, and azimuthal distribution for prestack azimuth processing.

Conclusions. Multicomponent OBC data are acquired with four components: a hydrophone and vertical geophone for the $P$-wavefield and two horizontal components, in-line and cross-line, for the $S$-wavefield. Although the vector-component coordinate system cannot be controlled during acquisition as well as in land surveys, $S$-wave data can be processed effectively within a source-centered coordinate system.

The single radial component that results from rotating the in-line and cross-line components into the source-receiver direction has important applications for lithology discrimination, imaging in the presence of gas, and depth imaging in high-velocity media.

$P$-$S$-wave two-component radial and transverse data can be reorganized into a different four-component system for fracture detection purposes. This vector coordinate system consists of four $S$-wave components in a source-and-receiver-centered coordinate system that allows for Alford rotation and layer stripping to unravel interference effects of $S$-wave splitting due to azimuthal anisotropy. Aside from enhanced $S$-waves for improved lithology discrimination, fracture properties can be determined for more accurate fracture models.

Suggestions for further reading.


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