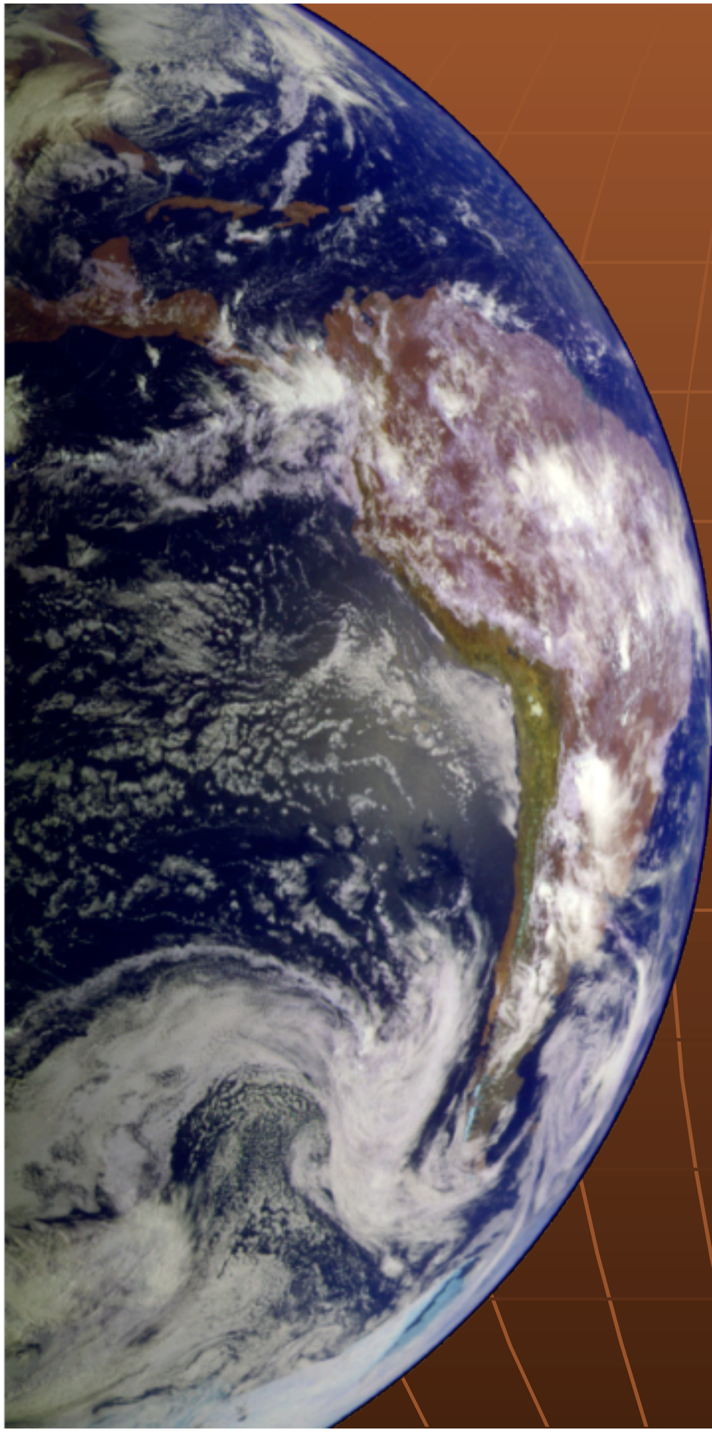


UT-AUSTIN EDGER FORUM

A Modified Boundary Element Method for Seismic Modeling

PhD Student: Yi Tao
Advisor: Mrinal Sen
February, 2010

THE UNIVERSITY OF TEXAS AT AUSTIN
JACKSON
SCHOOL OF GEOSCIENCES

Outline

- Introduction
- Modified Boundary Element Method
- Benchmark test
- Wave scattering in the frequency domain
- Synthetic seismogram examples
- Conclusions

Introduction

- Three major numerical simulation methods can be used for seismic modeling: FD (Finite Difference) , FEM (Finite Element Method) and BEM (Boundary Element Method)

- FD is the most widely used method in seismic modeling and migration.

- Strong impedance change among irregular interfaces, surface topographies and volume heterogeneities challenge the accuracy of FD.

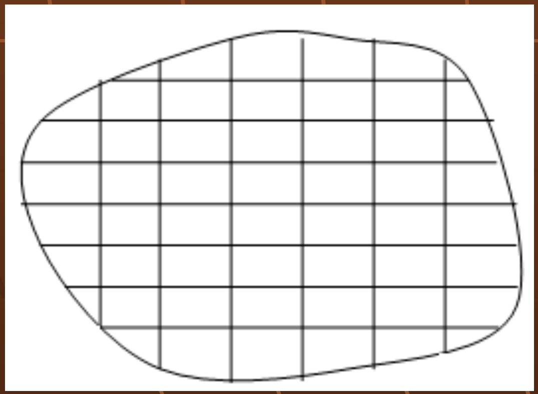
- FEM is another major method, compared to FD, it has the advantage of accuracy for the above geological structures.

Introduction

• BEM (Boundary Element Method) (Sanchez-Sesma, 1984, Bouchon, 1995, Fu, 2002) has the same advantage of accuracy of FEM.

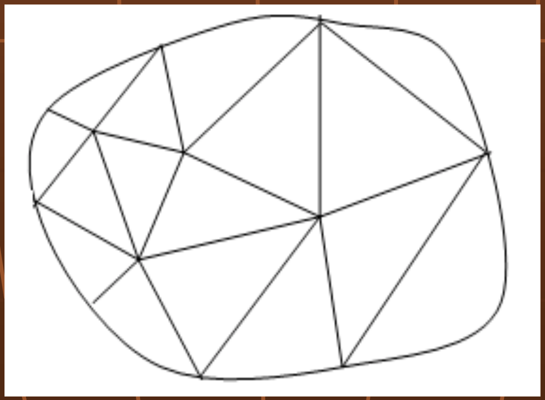
•Compared to FEM, BEM only requires grids on the boundary, has the advantage of less data preparation, but it requires that the media inside the domain is homogeneous.

Volume cells



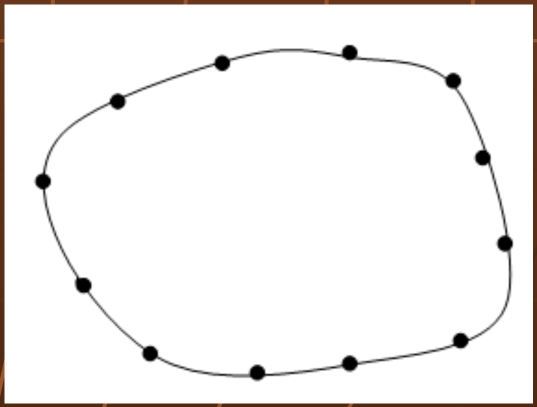
FD

Volume elements



FEM

Boundary elements



BEM

Introduction

Comparison of the three methods (Ordinary FD, Ordinary FEM, Direct BEM)

	FD	FEM	BEM
Domain	Whole volume	Whole volume	Boundary
Mesh	Regular mesh	Irregular mesh	Irregular mesh
Boundary conditions	Needs special treatment on curved structures	Free surface conditions automatically satisfied.	Free surface and inner interface conditions automatically satisfied.
Cost	Fast	Expensive (sparse global matrix)	Expensive (usually full global matrix)
Heterogeneity	OK	OK	Problematic

Introduction

- We use a modified BEM method which can simulate volume heterogeneities.

- Compared to BEM, it adds a volume scattering term. So it has all the advantages of BEM, only requires an additional storage of the volume scattering data.

- BEM can only be applied to piecewise heterogeneous media, if we know how to describe the scattering term, the modified method can be applied to general heterogeneous media.

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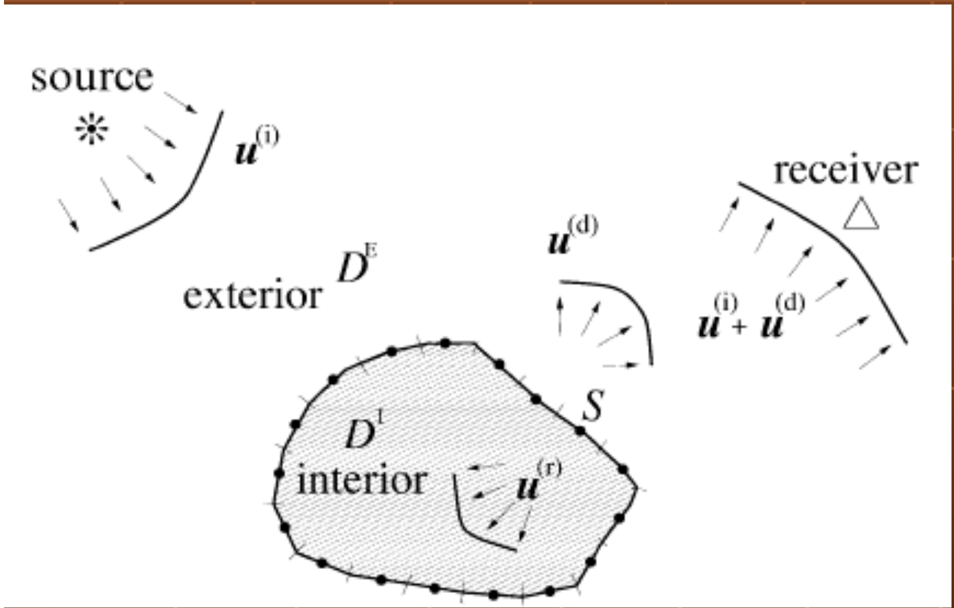
Modified Boundary Element Method



$$u(\mathbf{r}) = u^i(\mathbf{r}) + u^d(\mathbf{r})$$

The total wavefield recorded at a receiver which passes through a scatter can be written as the incident wavefield plus a scattered wavefield

Tim Pointer,et al, 1998

Modified Boundary Element Method

Acoustic wave Equation in the Frequency Domain:

$$\nabla^2 u(\mathbf{r}) + k(\mathbf{r})^2 u(\mathbf{r}) = -s(\mathbf{r},\omega)$$

Define a relative slowness perturbation

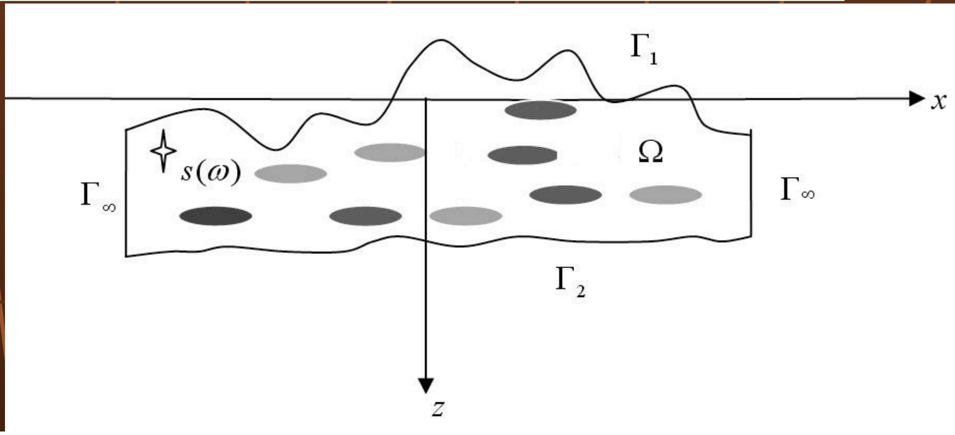
$$o(\mathbf{r}') = \rho(\mathbf{r})\mu_0(\mathbf{r}) / \rho_0(\mathbf{r})\mu(\mathbf{r}) - 1$$

$$\nabla^2 u(\mathbf{r}) + k_0(\mathbf{r})^2 u(\mathbf{r}) = -s(\mathbf{r},\omega) - k_0(\mathbf{r})^2 o(\mathbf{r})u(\mathbf{r})$$

Generalized Lippmann-Schwinger Equation

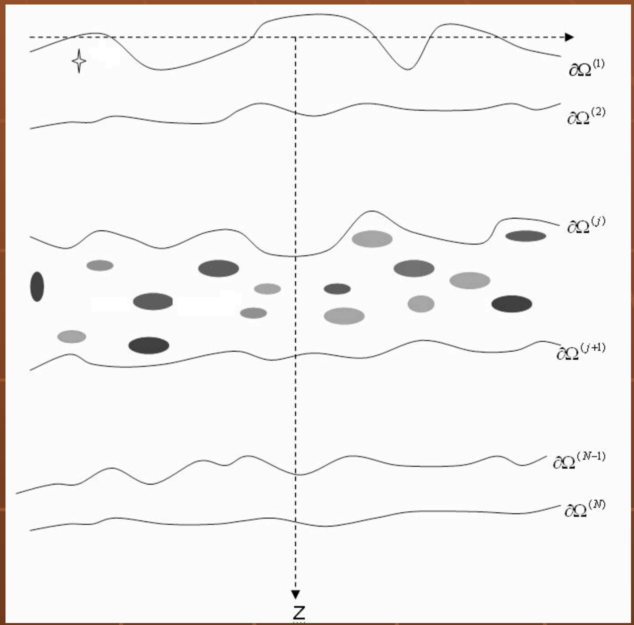
$$\int_\Gamma \left[G(\mathbf{r},\mathbf{r}') \frac{\partial u(\mathbf{r}')}{\partial n} - u(\mathbf{r}') \frac{\partial G(\mathbf{r},\mathbf{r}')}{\partial n} \right] d\mathbf{r}' + k_0^2 \int_\Omega o(\mathbf{r}')u(\mathbf{r}')G(\mathbf{r},\mathbf{r}')d\mathbf{r}' + S(\omega)G(\mathbf{r},\mathbf{r}_0)$$

$$= \begin{cases} u(\mathbf{r}) & \mathbf{r} \in \Omega \\ C(\mathbf{r})u(\mathbf{r}) & \mathbf{r} \in \Gamma \\ 0 & \mathbf{r} \notin \overline{\Omega} \end{cases}$$



Modified Boundary Element Method

Boundary conditions for layered media:



1.Free surface, the displacement is zero

$$\frac{\partial u(\mathbf{r})}{\partial n} = 0 \qquad \mathbf{r} \in \partial\Omega^{\text{£¨0£©}}$$

2.Inner interfaces: the displacement and traction are continuous along the interface

$$\begin{cases} u_-^{(j)}(\mathbf{r}) = u_+^{(j)}(\mathbf{r}) \\ \mu^{(j)} \dfrac{\partial u_-^{(j)}(\mathbf{r})}{\partial n} = \mu^{(j+1)} \dfrac{\partial u_+^{j}(\mathbf{r})}{\partial n} \end{cases} \quad \mathbf{r} \in \partial\Omega^{\text{£¨j£©}}$$

3. Infinite boundary: Sommerfield far field condition

$$\begin{cases} \lim\limits_{\mathbf{r}\to\infty} u(\mathbf{r}) = 0 \\ \lim\limits_{\mathbf{r}\to\infty} \dfrac{\partial u(\mathbf{r})}{\partial r} = iK_0 u(\mathbf{r}) \end{cases}$$

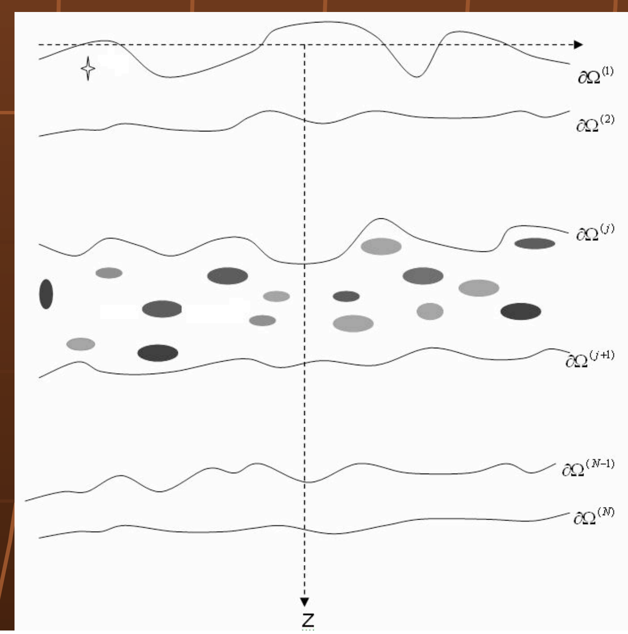
Modified Boundary Element Method

For layered media:

$$\int_{\partial\Omega^{(j)}}[G^{(j)}(\mathbf{r}',\omega)\frac{\partial u^{(j)}(\mathbf{r}')}{\partial n} - u^{(j)}(\mathbf{r}')\frac{\partial G^{(j)}(\mathbf{r},\mathbf{r}')}{\partial n}]d\mathbf{r}' - \int_{\partial\Omega^{(j+1)}}[G^{(j)}(\mathbf{r}',\omega)\frac{\partial u^{(j+1)}(\mathbf{r}')}{\partial n} - u^{(j+1)}(\mathbf{r}')\frac{\partial G^{(j)}(\mathbf{r},\mathbf{r}')}{\partial n}]d\mathbf{r}'$$

$$+[K_0^{(j)}]^2\int_{\Omega_s^{(j)}} O^{(j)}(\mathbf{r}')u^{(j)}(\mathbf{r}')G^{(j)}(\mathbf{r},\mathbf{r}')d\mathbf{r}' = \begin{cases} u^{(j)}(\mathbf{r}), & \mathbf{r}\in\Omega^{(j)} \\ C^{(j)}(\mathbf{r})u^{(j)}(\mathbf{r}), & \mathbf{r}\in\partial\Omega^{(j)} \\ C^{(j+1)}(\mathbf{r})u^{(j+1)}(\mathbf{r}), & \mathbf{r}\in\partial\Omega^{(j+1)} \\ 0, & \mathbf{r}\notin\overline{\Omega} \end{cases}$$

$o(\mathbf{r}') = 0$ **The media within a layer is homogeneous**



Modified Boundary Element Method

We use Green function in the background media

2D acoustic media:

$$G(\mathbf{r},\mathbf{r}_0) = \frac{i}{4} H_0^{(1)}(k|\mathbf{r}-\mathbf{r_0}|)$$

3D acoustic media:

$$G(\mathbf{r},\mathbf{r}_0) = \frac{1}{4\pi|\mathbf{r}-\mathbf{r_0}|}\exp(ik|\mathbf{r}-\mathbf{r_0}|)$$

Modified Boundary Element Method

Elastic media:

$$G_{ki}(\mathbf{r},\mathbf{r}_0) = A(U_1\delta_{ki} - U_2 r_{,k} r_{,i})$$

$$T_{ki} = \frac{\partial G_{ki}(\mathbf{r},\mathbf{r}_0)}{\partial n} = \mu A \left\{ \left[\pounds \;\; \delta_{ki} + \pounds \frac{\partial r}{\partial n} \oplus \ddot{}\, n_k r_{,k} \qquad \frac{\lambda}{\mu} n_i r_{,k} \right] \frac{\partial U_1}{\partial r} \right.$$

$$- \left[\pounds \;\; \delta_{ki} + \pounds \frac{\partial r}{\partial n} \oplus 2 i\!\!/\, n_k r_{,i} \qquad n_i r_{,k} - 2 r_{,k} r_{,i} \frac{\partial r}{\partial n}) + a \frac{\lambda}{\mu} + n_i r_{,k} \right] \frac{U_2}{r}$$

$$\left. - \left[2 r_{,k} r_{,i} \frac{\partial r}{\partial n} + \frac{\lambda}{\mu} + n_i r_{,k} \right] \frac{U_2}{r} \right\}$$

2D elastic:

$$A = \frac{i}{4\mu}$$

$$U_1 = H_0^{(1)}(k_\beta r) - \frac{1}{k_\beta r} H_1^{(1)}(k_\beta r) + \left(\frac{k_\alpha}{k_\beta}\right)^2 \frac{1}{k_\alpha r} H_1^{(1)}(k_\alpha r)$$

$$U_2 = -H_0^{(1)}(k_\beta r) + \left(\frac{k_\alpha}{k_\beta}\right)^2 \frac{1}{k_\alpha r} H_1^{(1)}(k_\alpha r)$$

3D elastic:

$$A = \frac{1}{4\pi\mu}$$

$$U_1 = \frac{e^{ik_\beta r}}{r} + \left[\frac{i}{k_\beta r} - \left(\frac{1}{k_\beta r}\right)^2 \right] \frac{e^{ik_\beta r}}{r} - \left(\frac{k_\alpha}{k_\beta}\right)^2 \left[\frac{i}{k_a r} - \left(\frac{1}{k_a r}\right)^2 \right] \frac{e^{ik_\alpha r}}{r}$$

$$U_2 = \left[1 + \frac{3i}{k_\beta r} - 3\left(\frac{1}{k_\beta r}\right)^2 \right] \frac{e^{ik_\beta r}}{r} - \left(\frac{k_\alpha}{k_\beta}\right)^2 \left[1 + \frac{3i}{k_a r} - 3\left(\frac{1}{k_a r}\right)^2 \right] \frac{e^{ik_\alpha r}}{r}$$

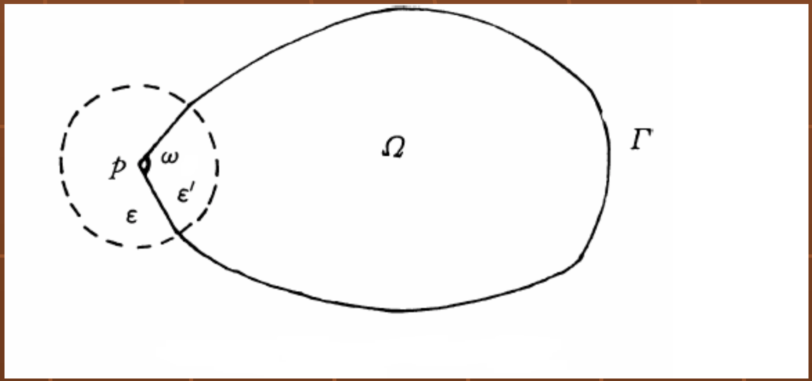
Modified Boundary Element Method

The coefficient C(r) in **acoustic media**:

Smooth points: C(r)＝0.5

2-D non-smooth points:

$$C(\mathbf{r}) = \frac{\omega}{2\pi}$$



3-D non-smooth points :

$$C(\mathbf{r}) = \frac{\omega}{4\pi}$$

Solid angle

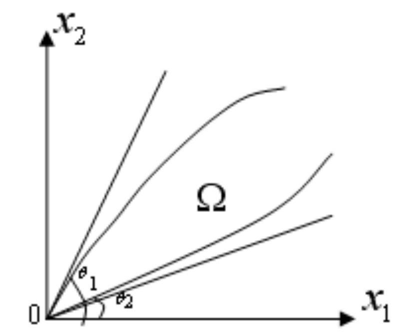
$$\omega = \frac{\arccos(\cos\alpha - \cos\beta\cos\gamma)}{\sin\beta\sin\gamma} + \frac{\arccos(\cos\beta - \cos\alpha\cos\gamma)}{\sin\alpha\sin\gamma} + \frac{\arccos(\cos\gamma - \cos\alpha\cos\beta)}{\sin\alpha\sin\beta}$$

Modified Boundary Element Method

The coefficient C(r) in **elastic media**:

Smooth points: C(r)＝0.5

2-D non-smooth points:



$$c(\mathbf{r}) = \frac{1}{4\pi(1-\nu)}\begin{bmatrix} 2(1-\nu)(\theta_2-\theta_1)+0.5(\sin^2(2\theta_2)-\sin^2(2\theta_1)) & \sin^2\theta_2-\sin\theta_1 \\ sym & 2(1-\nu)(\theta_2-\theta_1)-0.5(\sin^2(2\theta_2)-\sin^2(2\theta_1)) \end{bmatrix}$$

3-D non-smooth points :

$$c(\mathbf{r}) = \frac{1}{8\pi(1-\nu)}\begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix}$$

$$C_{11} = \sin(2\theta_2)-\sin(2\theta_1)+4(1-\nu)(\theta_2-\theta_1)$$

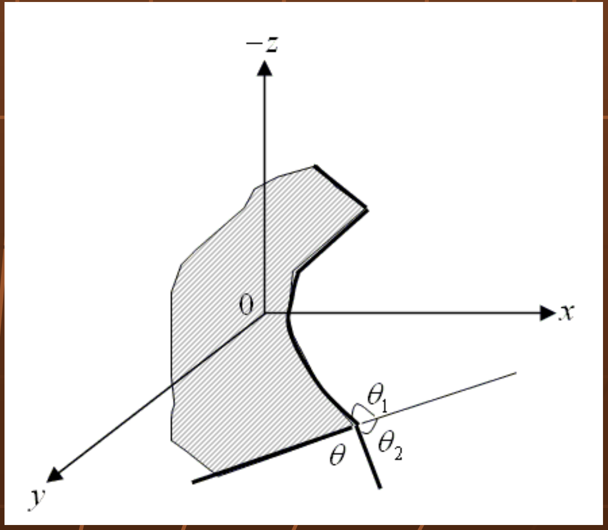
$$C_{22} = -\sin(2\theta_2)+\sin(2\theta_1)+4(1-\nu)(\theta_2-\theta_1)$$

$$C_{33} = 4(1-\nu)(\theta_2-\theta_1)$$

$$C_{12} = C_{21} = 2(\sin^2(\theta_2)-\sin^2(\theta_1))$$

$$C_{13} = C_{31} = 0$$

$$C_{23} = C_{32} = 0$$



Modified Boundary Element Method

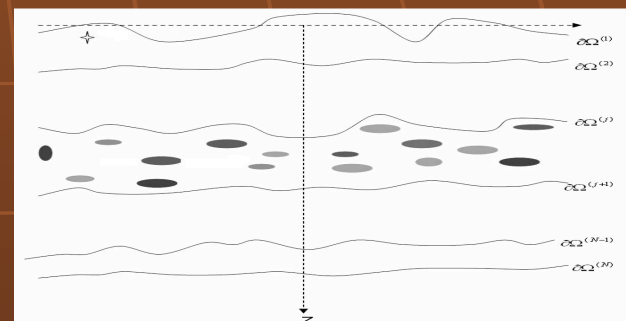
Numerical implementation of the method

Boundary scattering (BEM):

$$H^{(i,1)}\mathbf{u}^{(i)}(\mathbf{r}_j) - G^{(i,1)}\mathbf{t}^{(i)}(\mathbf{r}_j) = H^{i,2}\mathbf{u}^{(i-1)}(\mathbf{r}_j) + G^{i,2}\mathbf{t}^{(i-1)}(\mathbf{r}_j)$$

Volume scattering :

$$K^{(i)}\mathbf{w}^{(i)}(\mathbf{r}_j)$$



$$\mathbf{A}_1^{(i)}\mathbf{Q}^{(i)} + \mathbf{A}_2^{(i)}\mathbf{Q}^{(i+1)} + \mathbf{K}^{(i)}w^{(i)} = \delta_{si}\mathbf{s}$$

$$\begin{bmatrix} \mathbf{A}_1^{(N+1)} & & & & & & & & & \\ \mathbf{A}_2^{(N)} & \mathbf{K}^{(N)} & \mathbf{A}_1^{(N)} & & & & & & & \\ & \vdots & \vdots & \vdots & & & & & & \\ & & \mathbf{A}_2^{(i+1)} & \mathbf{K}^{(i+1)} & \mathbf{A}_1^{(i+1)} & & & & & \\ & & & & \mathbf{A}_2^{(i)} & \mathbf{K}^{(i)} & \mathbf{A}_1^{(i)} & & & \\ & & & & & \vdots & \vdots & \vdots & & \\ & & & & & & \mathbf{A}_2^{(2)} & \mathbf{K}^{(2)} & \mathbf{A}_1^{(2)} & \\ & & & & & & & \mathbf{A}_2^{(1)} & \mathbf{K}^{(1)} & \mathbf{A}_1^{(1)} \end{bmatrix} \begin{bmatrix} m^{(N+1)} \\ m^{(N)} \\ \vdots \\ m^{(i+1)} \\ m^{(i)} \\ \vdots \\ m^{(2)} \\ m^{(1)} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ s(\omega) \end{bmatrix}$$

For layered media, we have a sparse global matrix

Modified Boundary Element Method

Define the boundaries, form modified boundary-volume integral equation

↓

Numerically describe the boundaries and the volume (elements and shape functions)

↓

Form a global matrix, solve it to get wavefield at each boundary in frequency domain

↓

Put the wavefield back to the equation to get wavefield at the receiver line

↓

Inverse FFT

↓

Synthetic seismic data

Workflow of the method

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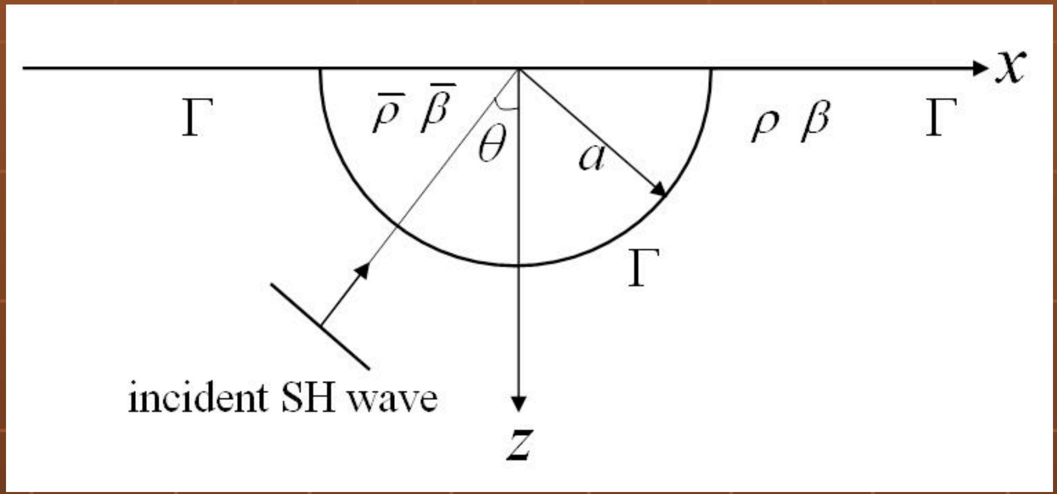
Benchmark test



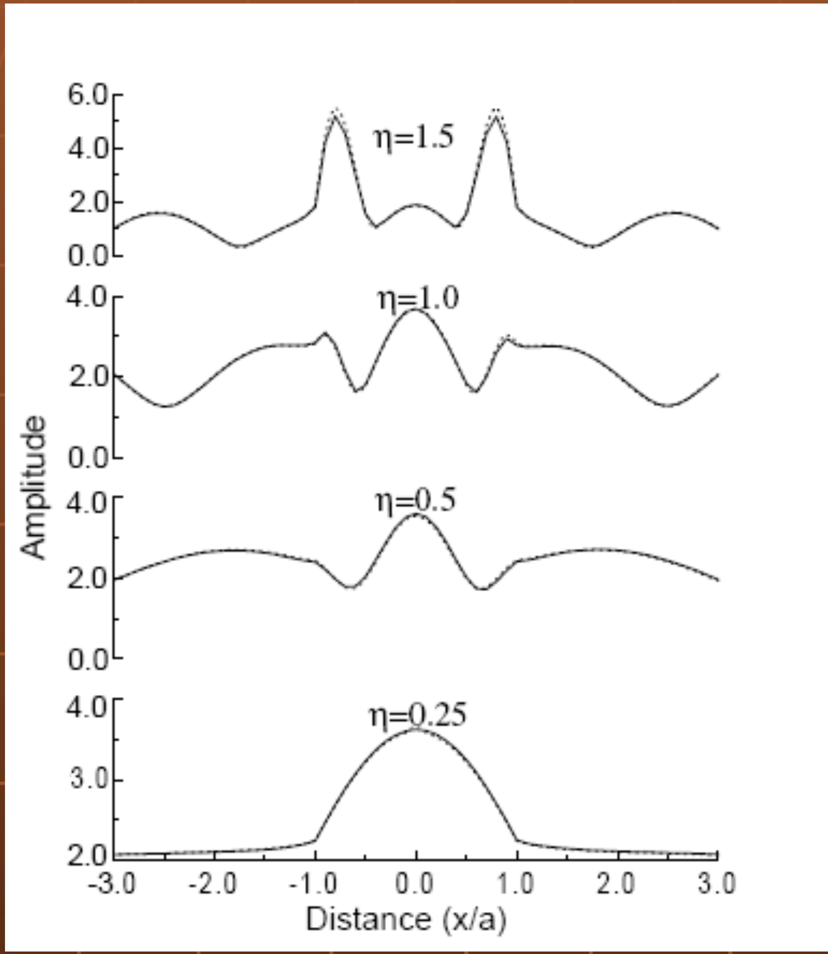
The valley model is first given by M.D.Trifunac(1973)

SH or acoustic wave

Dimensionless frequency： $\eta = \frac{2a}{\lambda} = \frac{a\omega}{\pi\beta}$

$\rho / \bar{\rho} = 1.5$ $\beta / \bar{\beta} = 2$

Benchmark test

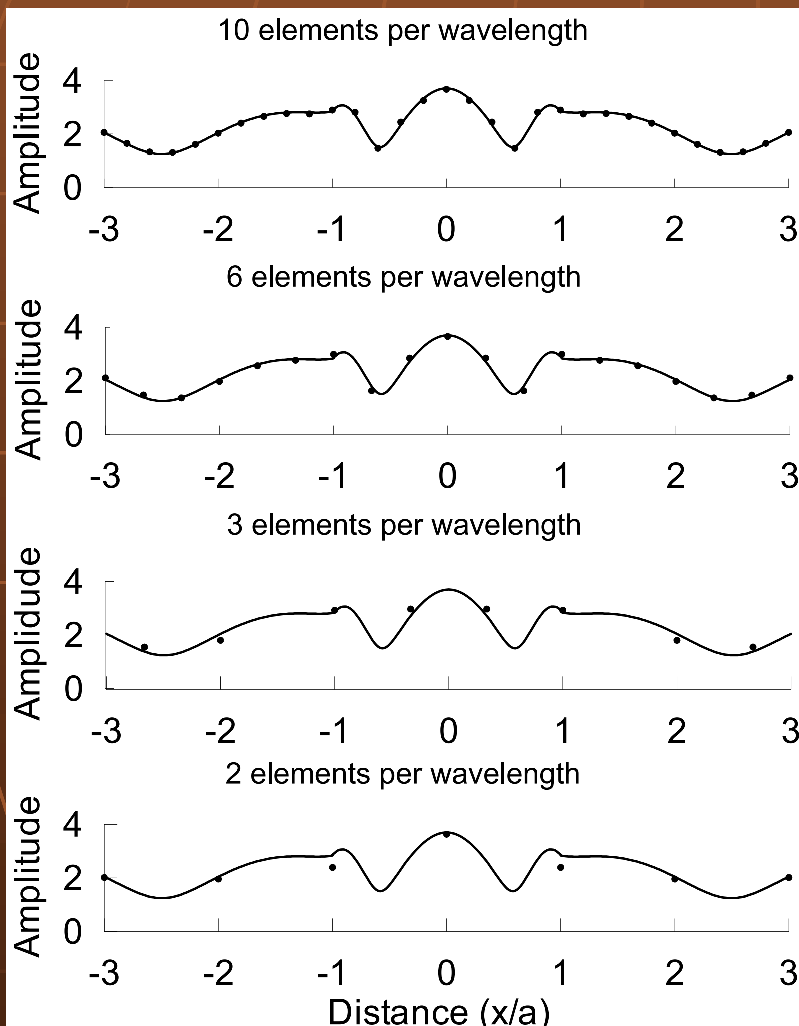


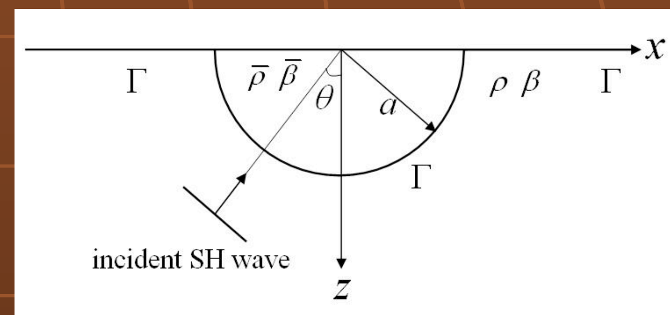
Seismic response in the frequency domain (normal incidence)

$\eta = 1$

Benchmark test

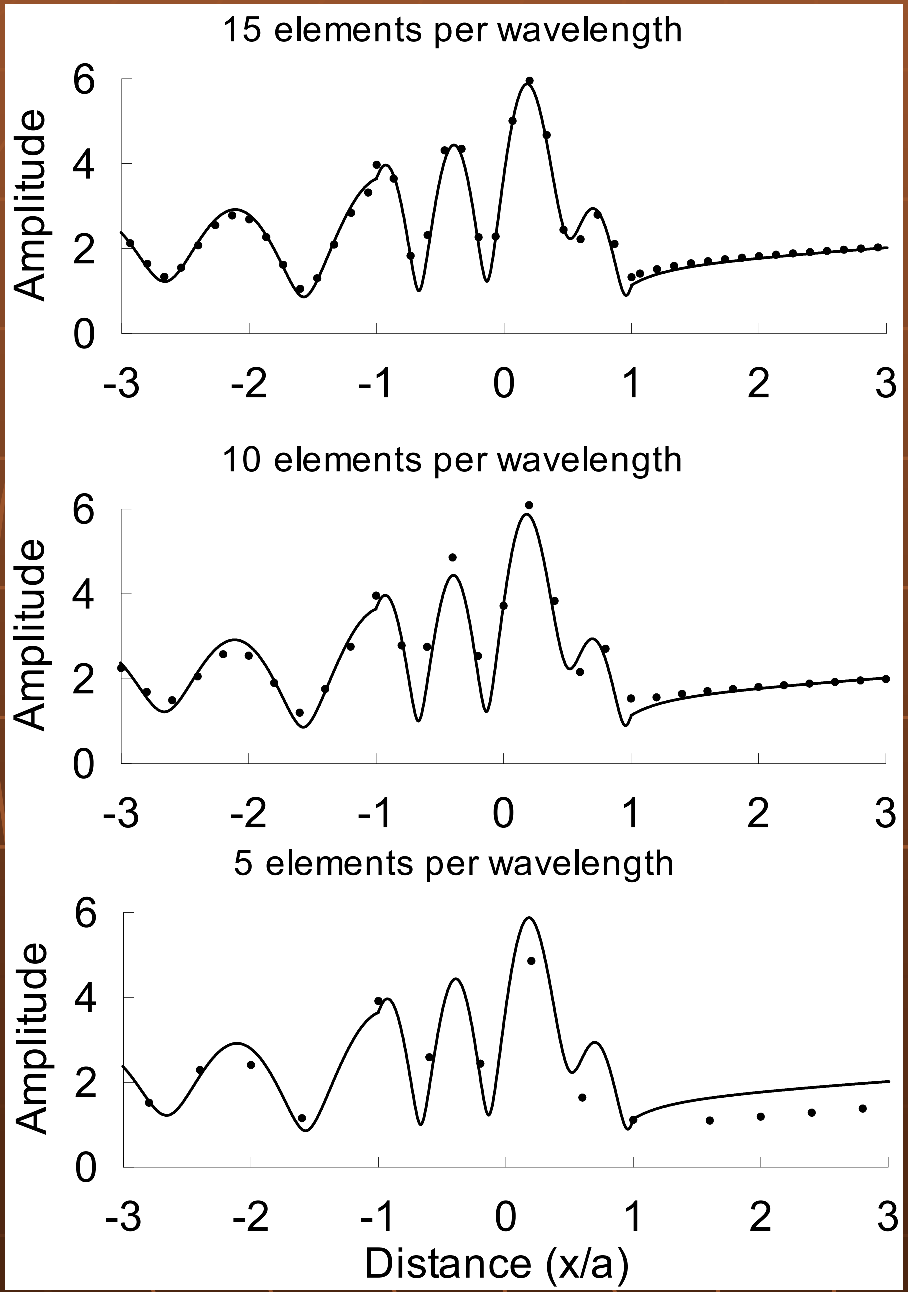
How many elements per wavelength do we need for seismic wave simulation?





Vertical incidence:

three elements per wavelength may be enough

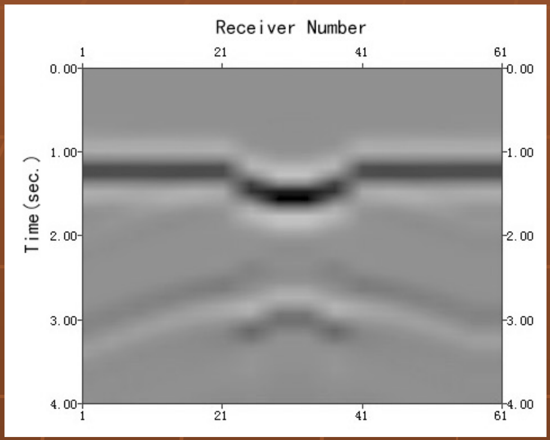


60° **incidence:**

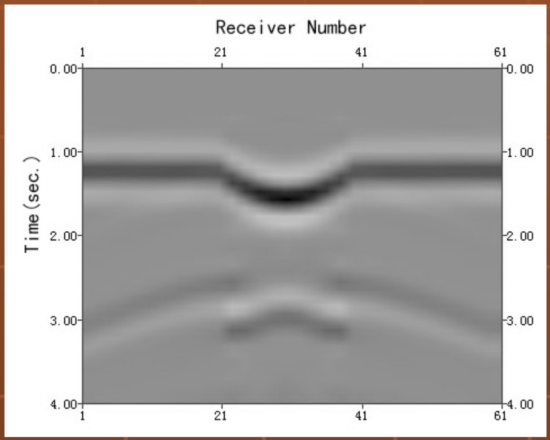
Five elements per wavelength still has some errors

If the geological structure is complex and we want to be more accurate, we should use more elements per wavelength

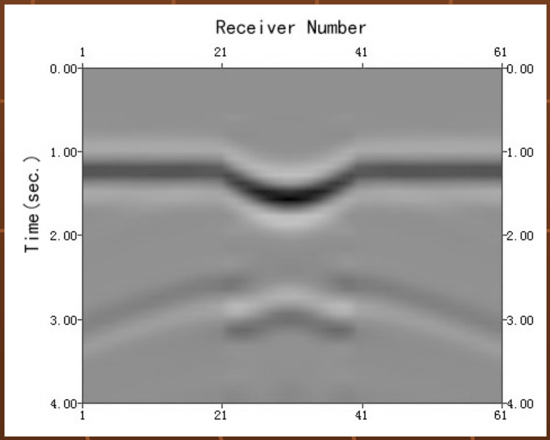
Benchmark test



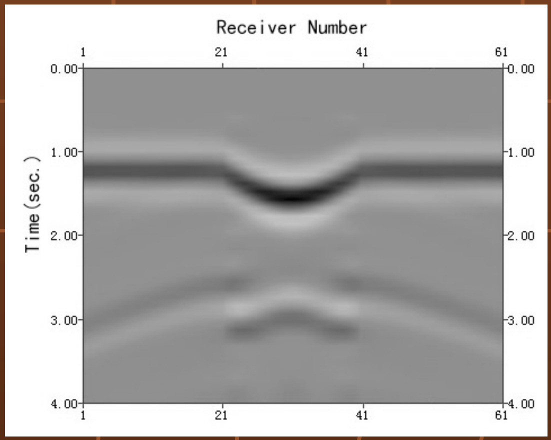
30 elements per wavelength



10 elements per wavelength



3 elements per wavelength



2 elements per wavelength

We see little difference from the synthetic data in the time domain

Benchmark test

- In the frequency domain, we may have better chance to see the sensitivity of the parameters of the model.

- Seismic response in frequency domain is used to study ground motion in earthquake engineering.

- Before generating synthetic seismograms, we first do some analysis in the frequency domain.

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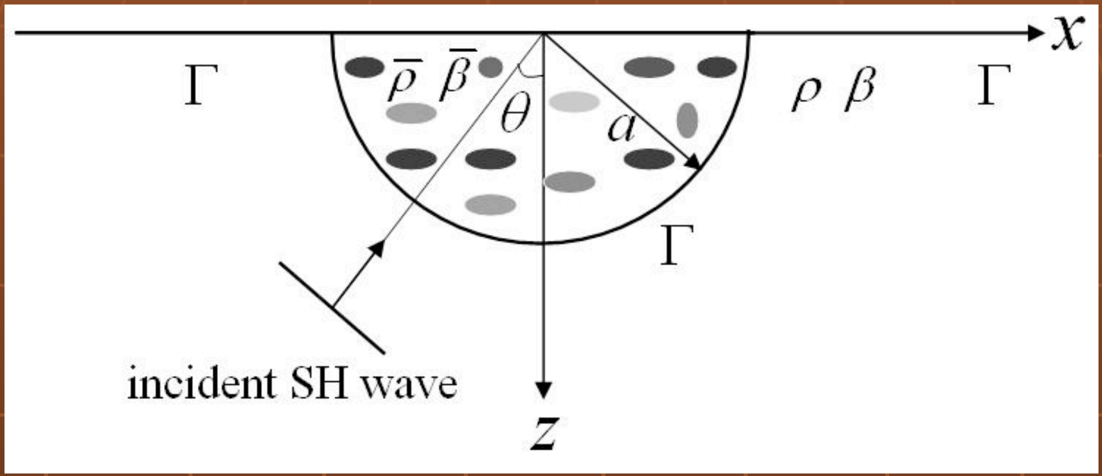
Wave scattering in the frequency domain



Modified valley model: random media in the valley

$$v(\mathbf{r}) = v_0(\mathbf{r}) + \delta v(\mathbf{r})$$

Wave scattering in the frequency domain

Six different random media

Different types of random media	Filtering factor
Gaussian	$\hat{f}_G(k) = \kappa \exp(-\frac{a_G^2 k^2}{8})$
Exponential	$\hat{f}_e(k) = \kappa[a^{-2} + k^2]^{-\frac{d+1}{4}}$
Von-Karman	$\hat{f}_K(k) = \kappa[a^{-2} + k^2]^{-\frac{d}{4}-\frac{N}{2}}$
Flicker noise	$\hat{f}_f(k) = \kappa k^{-1}$
Brown noise	$\hat{f}_b(k) = \kappa k^{-3/2}$
White noise	$\hat{f}_w(k) = \kappa$

a :autocorrelation length

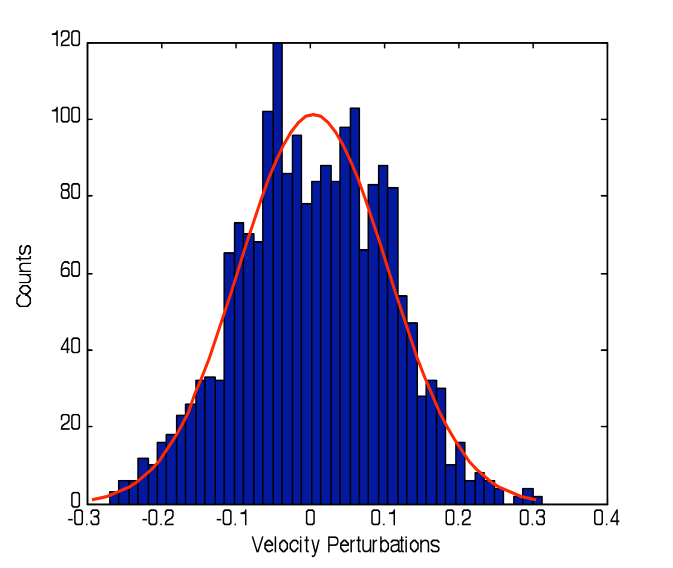
N:Hurst index

k:wavenumber

constant

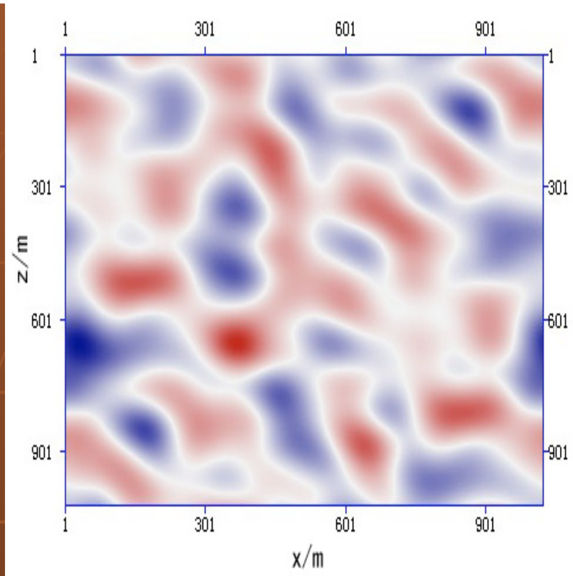
Wave scattering in the frequency domain

Gaussian type random media

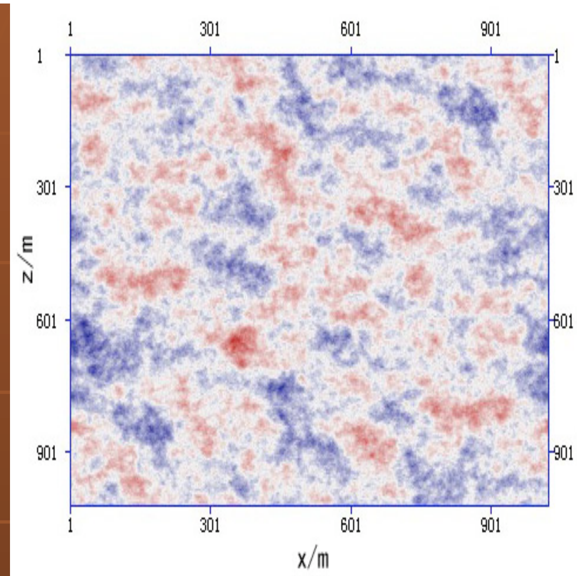


a=2.5m,   δ=0.1

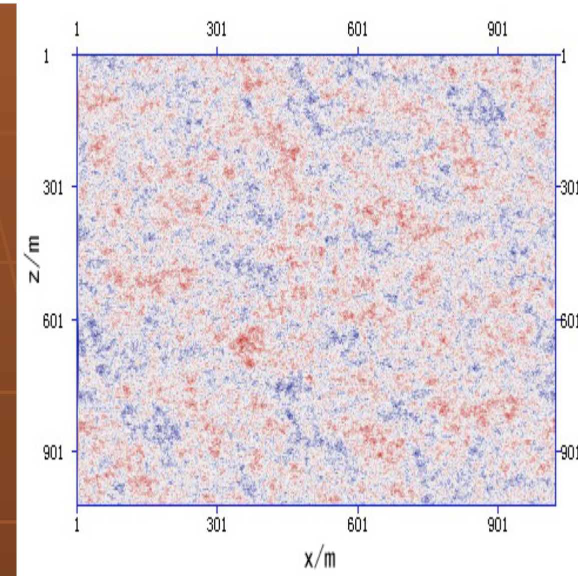
Although there are some deviations, the random media is generally assumed to satisfy statistical rules.



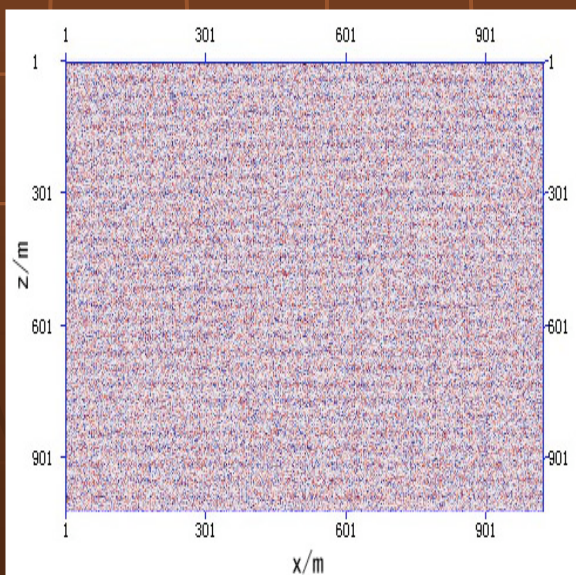
Gaussian



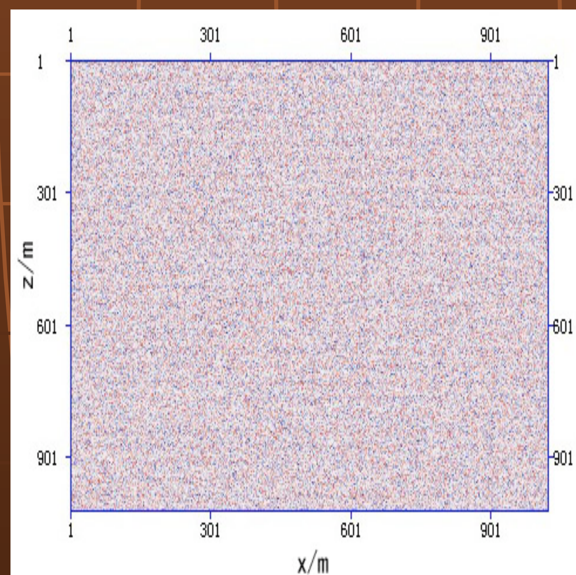
Exponential



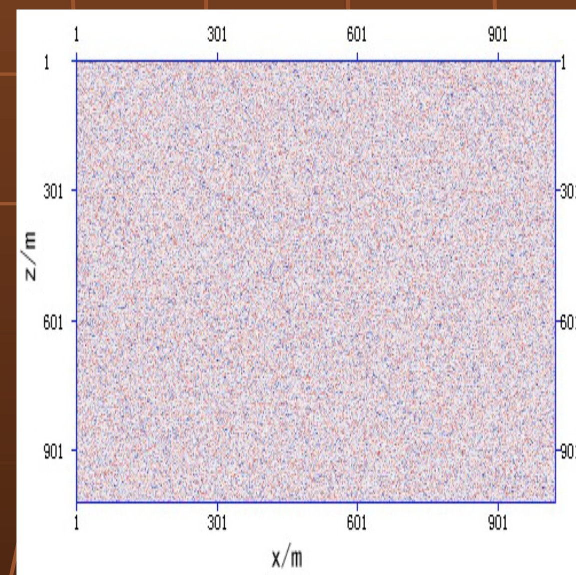
Von-Karman



Flicker noise

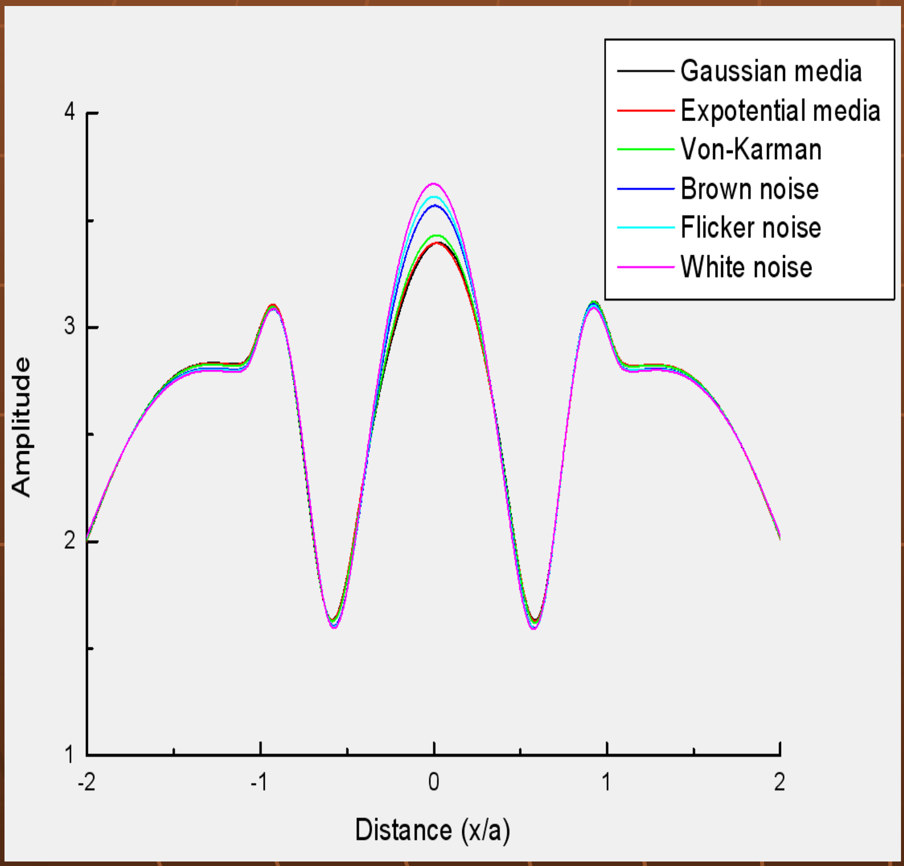


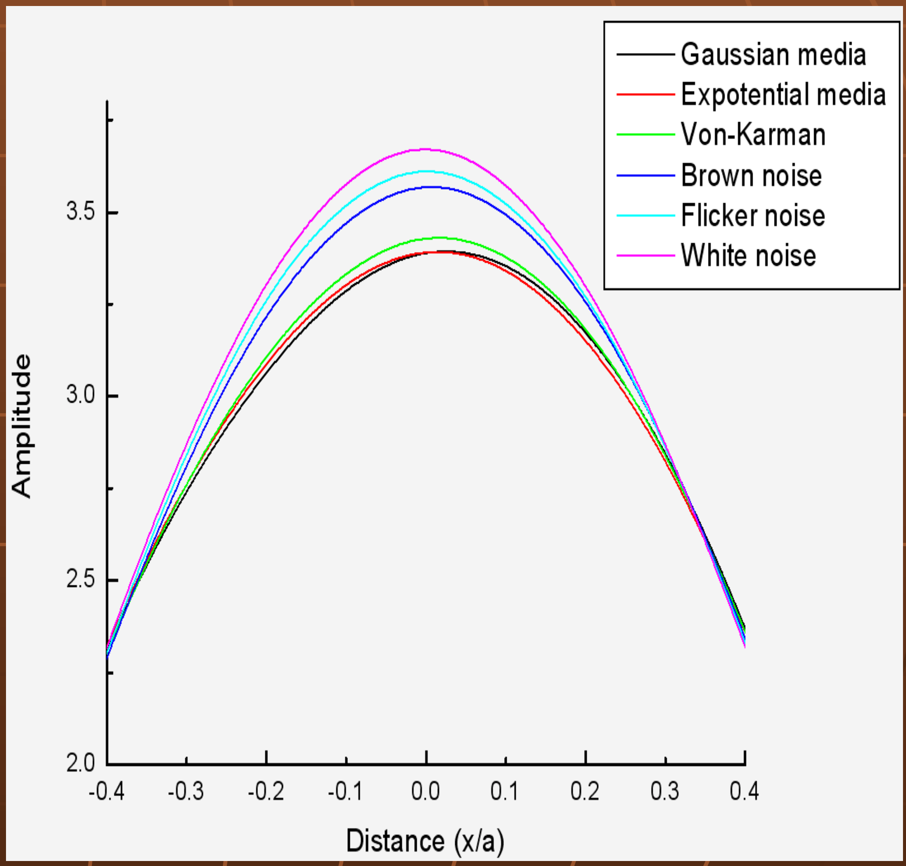
Brown noise



White noise

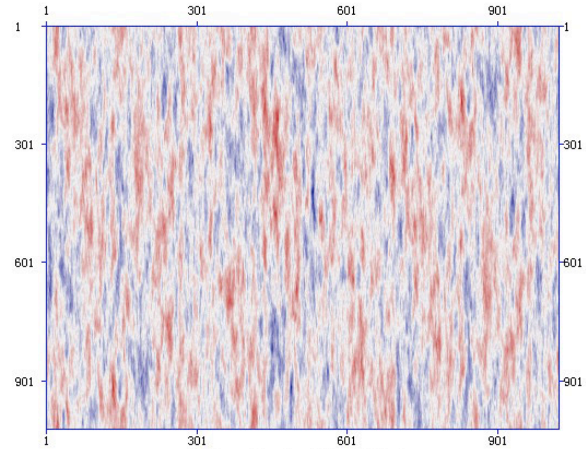
Wave scattering in the frequency domain





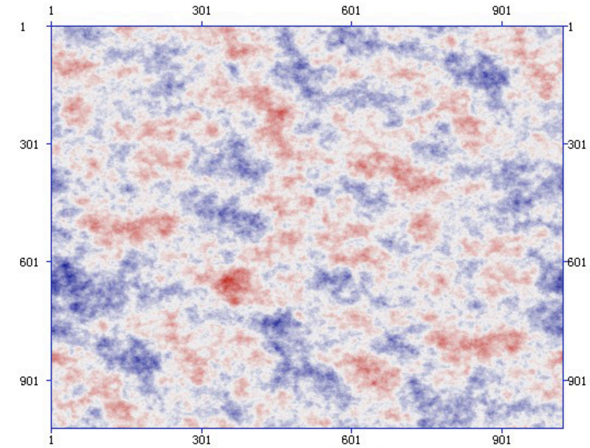
Frequency response with different types of random media

Different horizontal correlation length:
R= horizontal correlation length/vertical correlation length



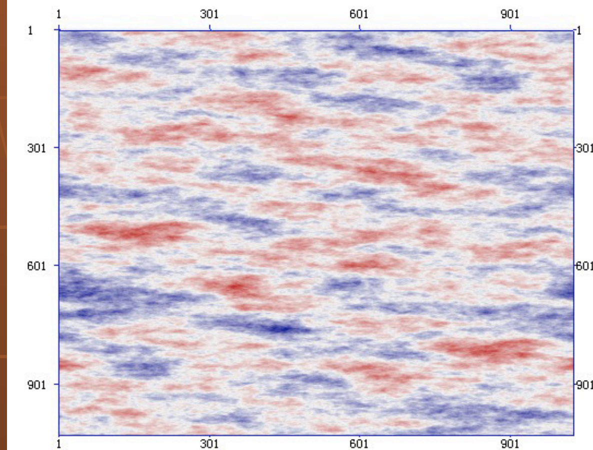
（1） a=10m, b=100m, R=0. 1

R＝0.1



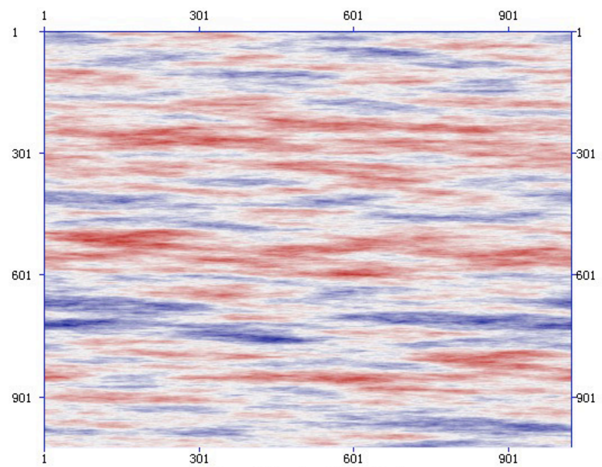
（2） a=100m, b=100m, R=1

R＝1



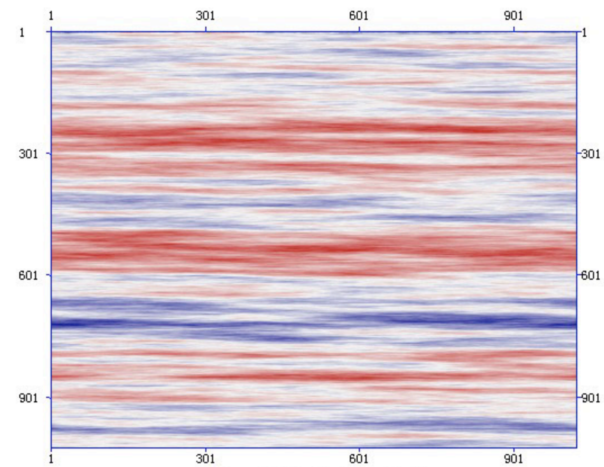
（3） a=300m, b=100m, R=3

R＝3



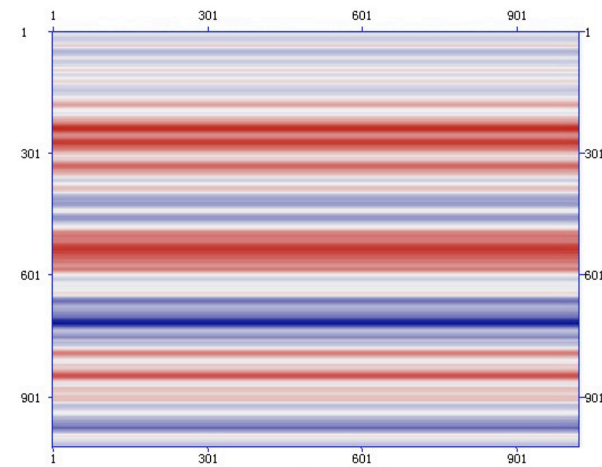
（4） a=800m, b=100m, R=8

R＝8



（5） a=2000m, b=100m, R=20

R＝20

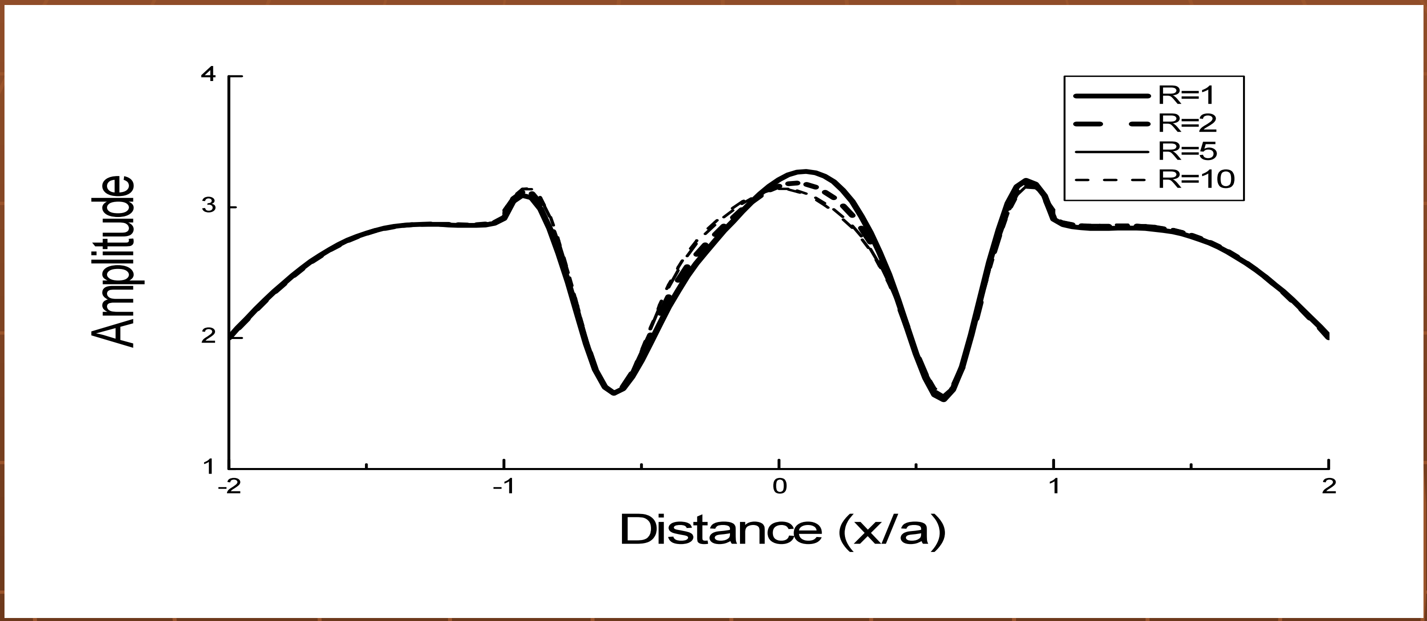


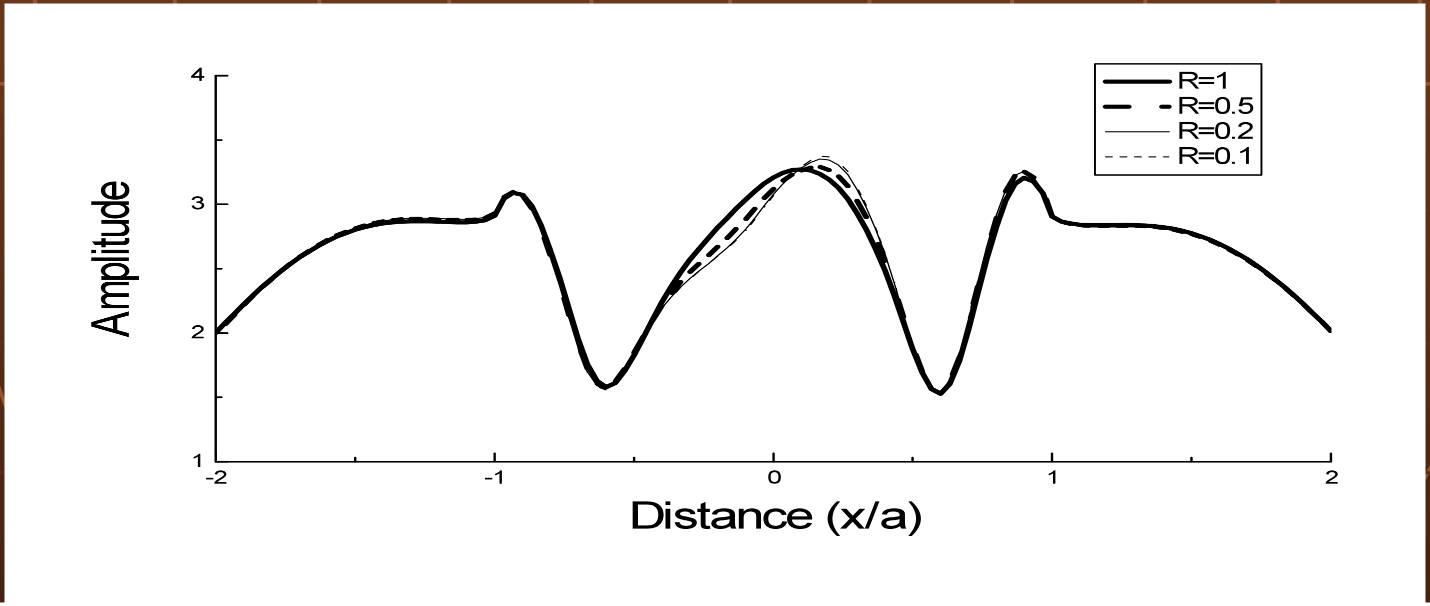
（6） a=+∞, b=100m, R=∞

R＝∞

Wave scattering in the frequency domain

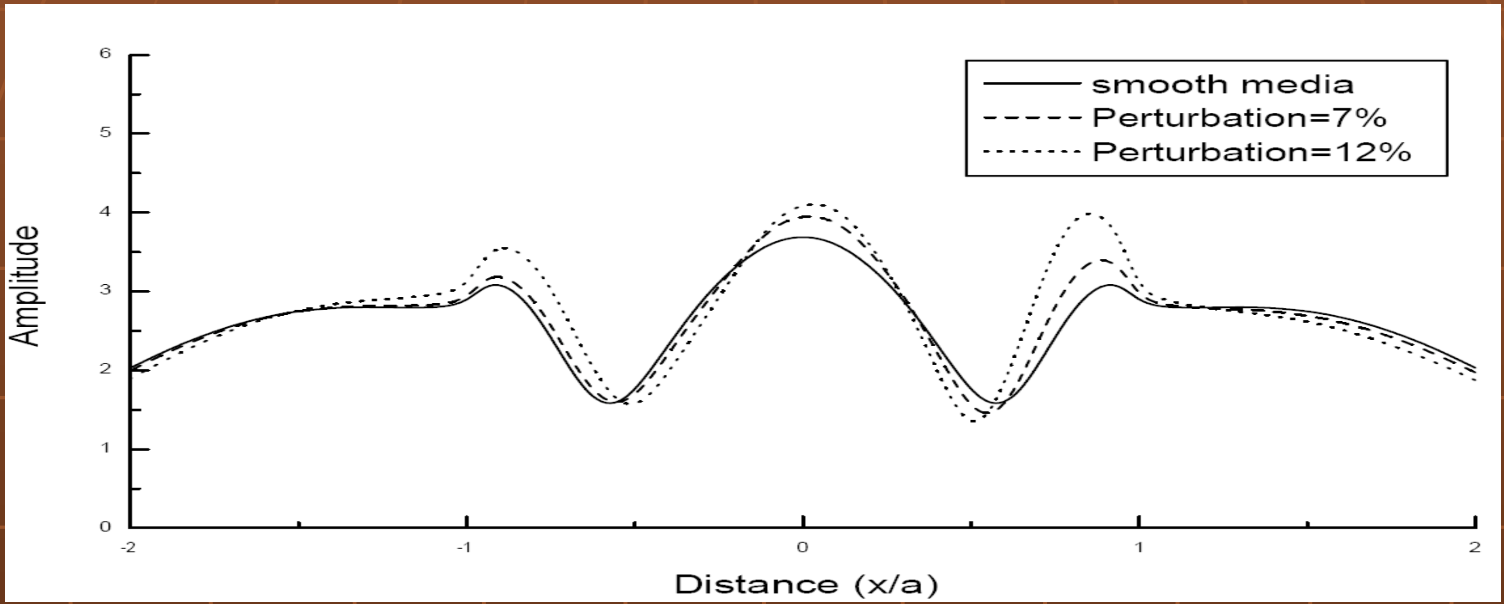
Frequency response with different correlation lengths

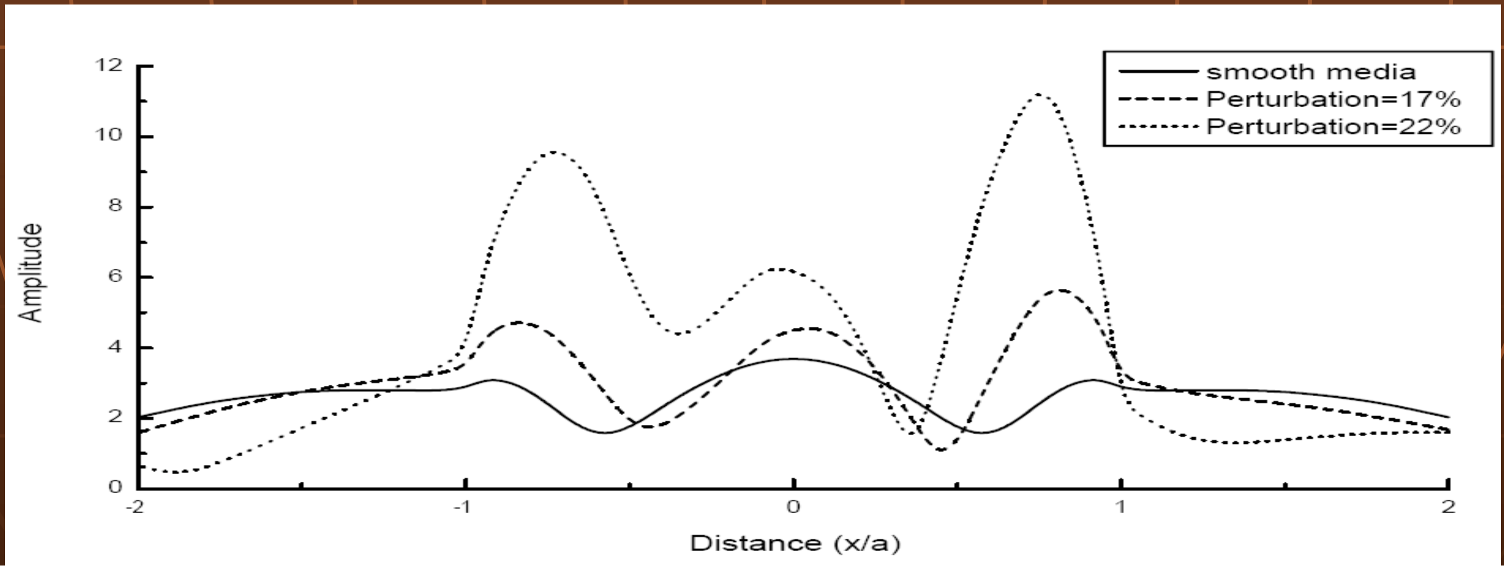




Wave scattering in the frequency domain

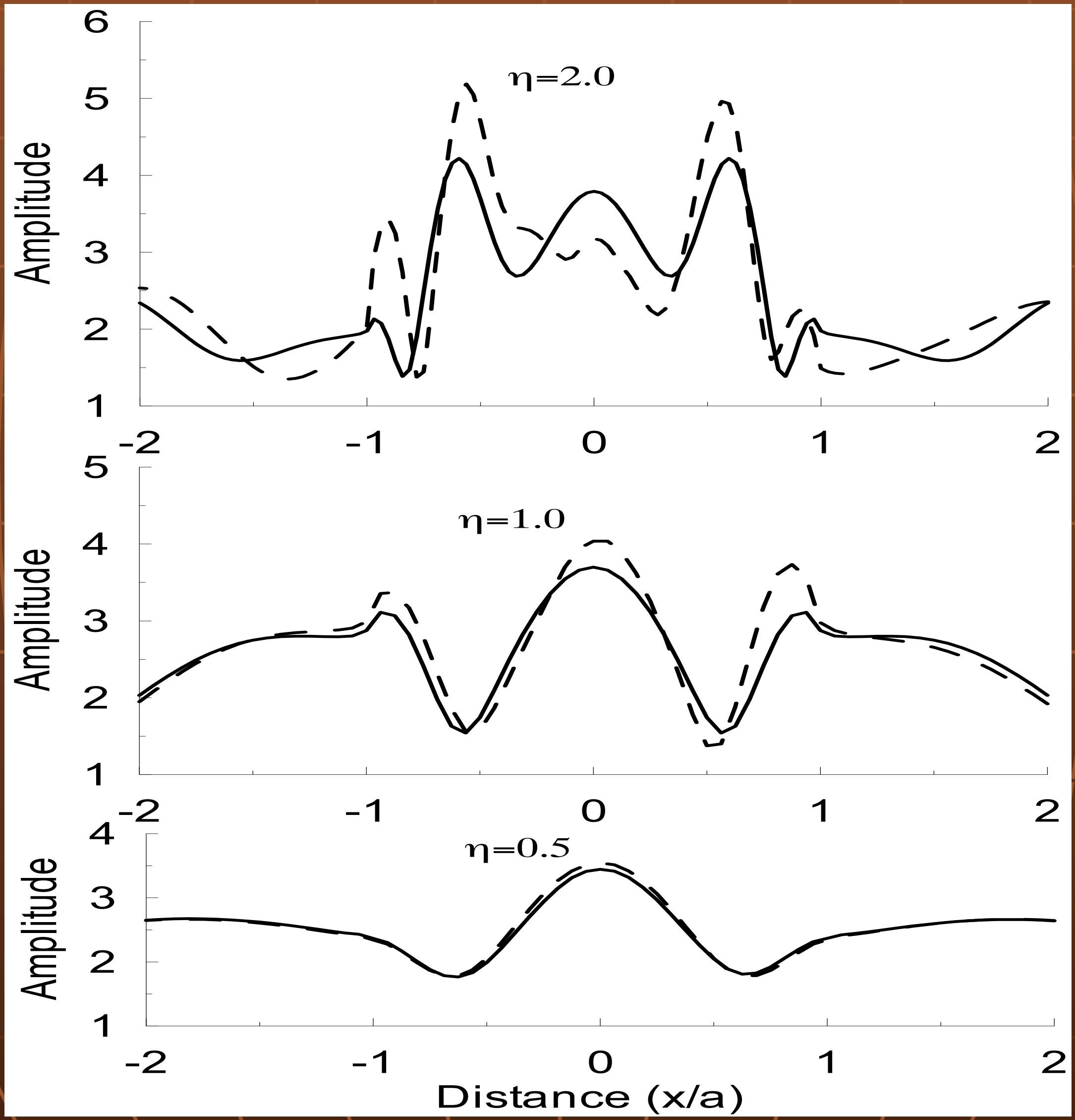
Different Perturbations





Wave scattering in the frequency domain

Seismic response for different dimensionless frequencies



Wave scattering in the frequency domain

- Different types of random media and different parameters of the random media have different effect on the results.

- Perturbations and dimensionless frequency (the complexity of the structure) affects the results much bigger than others.

- The analysis allows us to better choose the parameters of the random media.

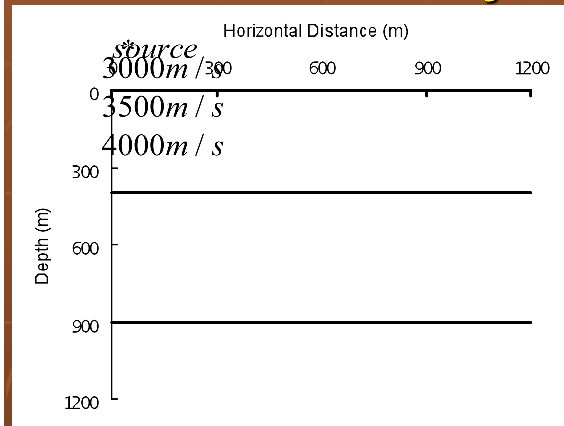
Outline

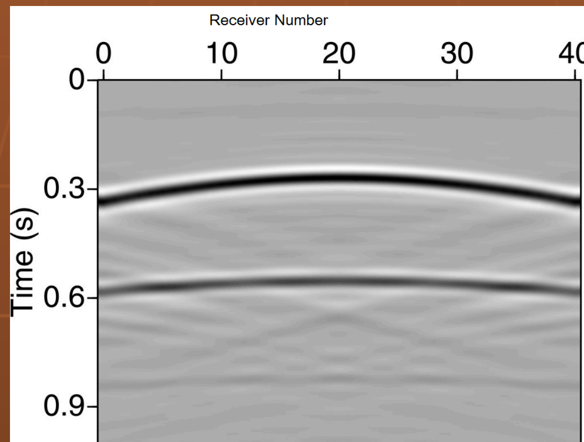
- Introduction
- Modified Boundary Element Method
- Benchmark test
- Wave scattering in the frequency domain
- Synthetic seismogram examples
- Conclusions

Synthetic seismogram examples

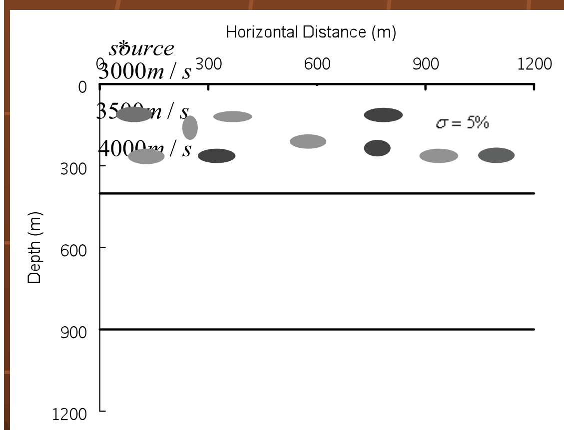
Random media scattering

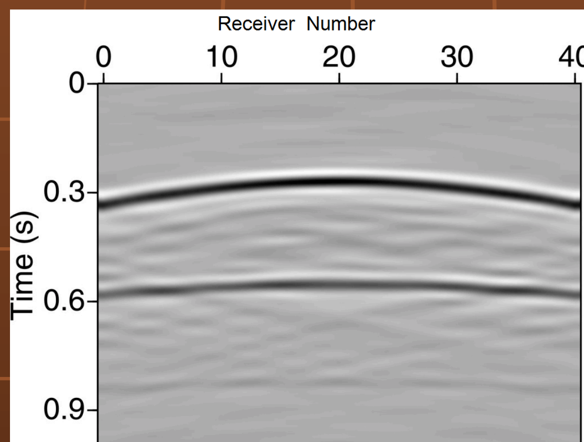
No perturbation



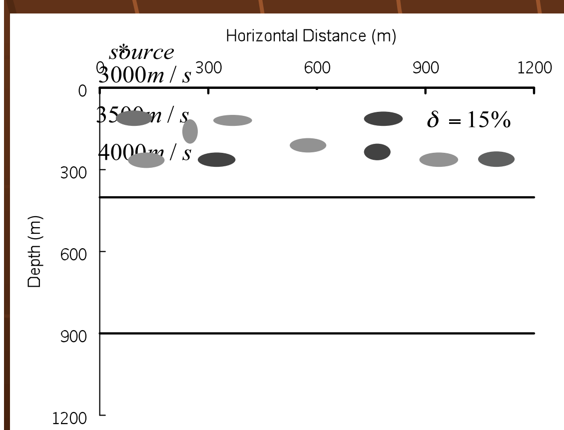


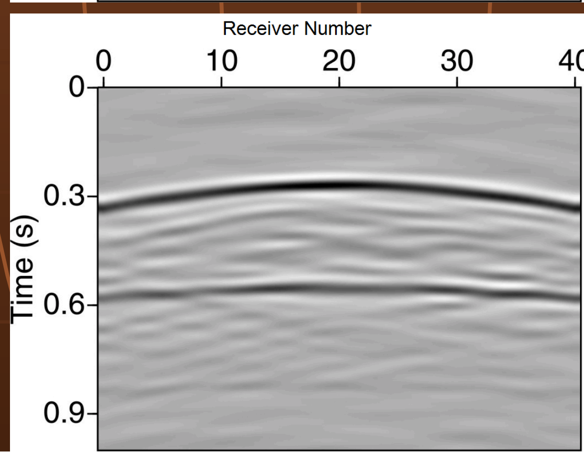
Perturbation=5%



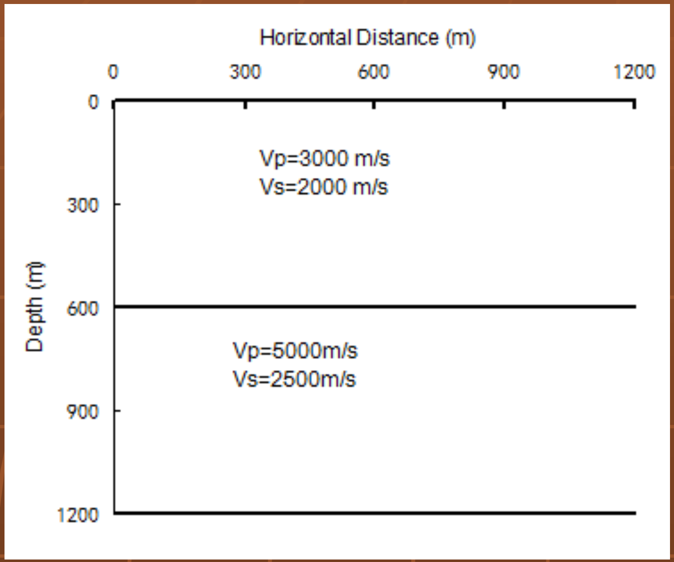


Perturbation=15%

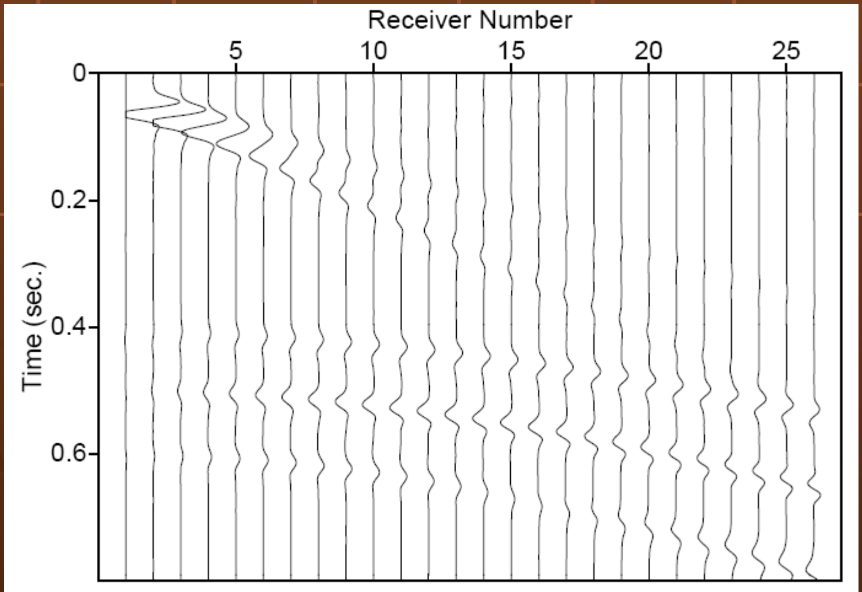




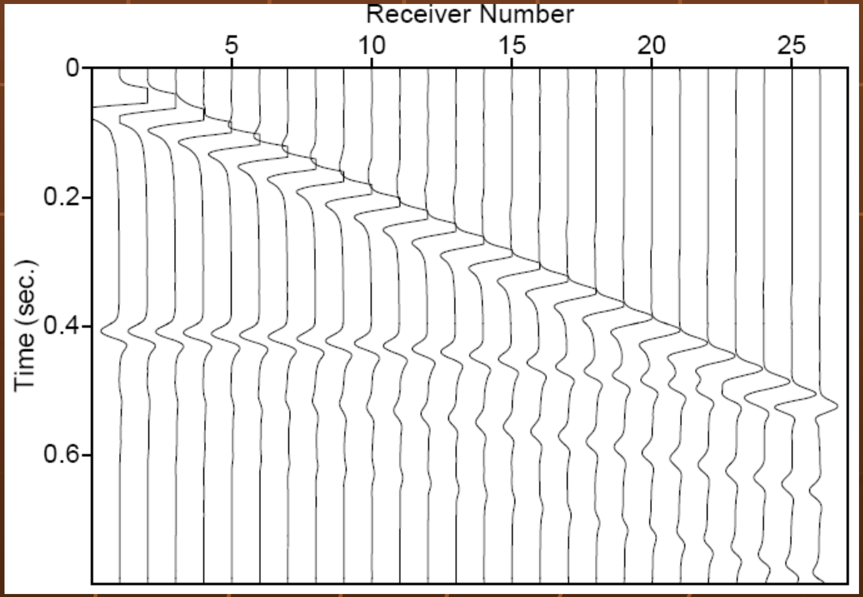
Synthetic seismogram examples



Simple 2-D elastic model for test



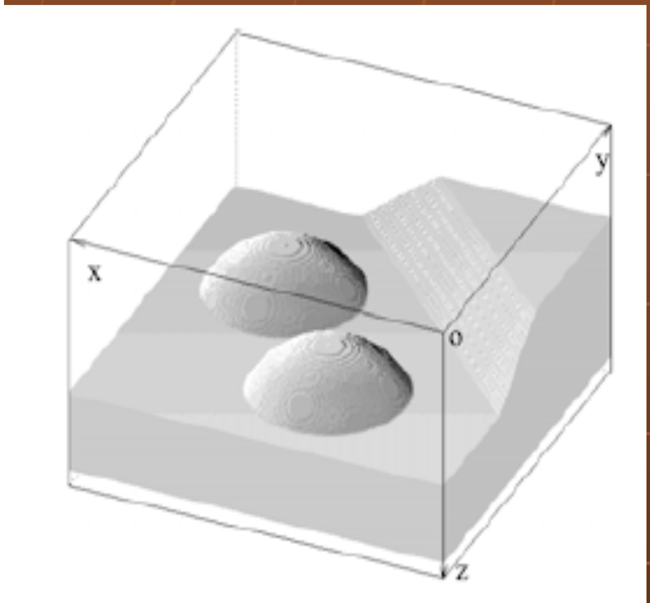
x component

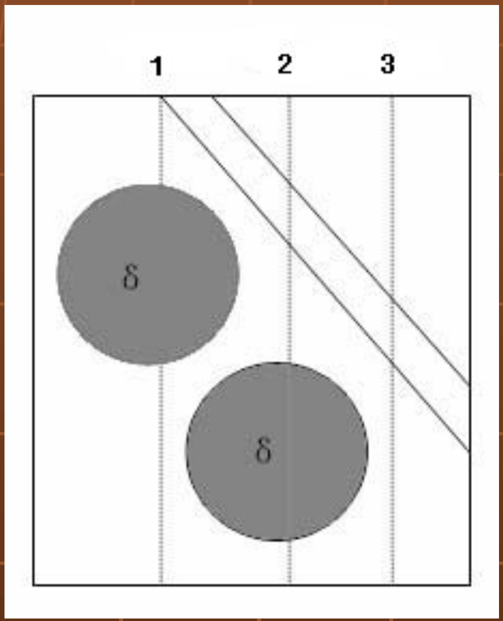


z component

Synthetic seismogram examples

3D example: French Model



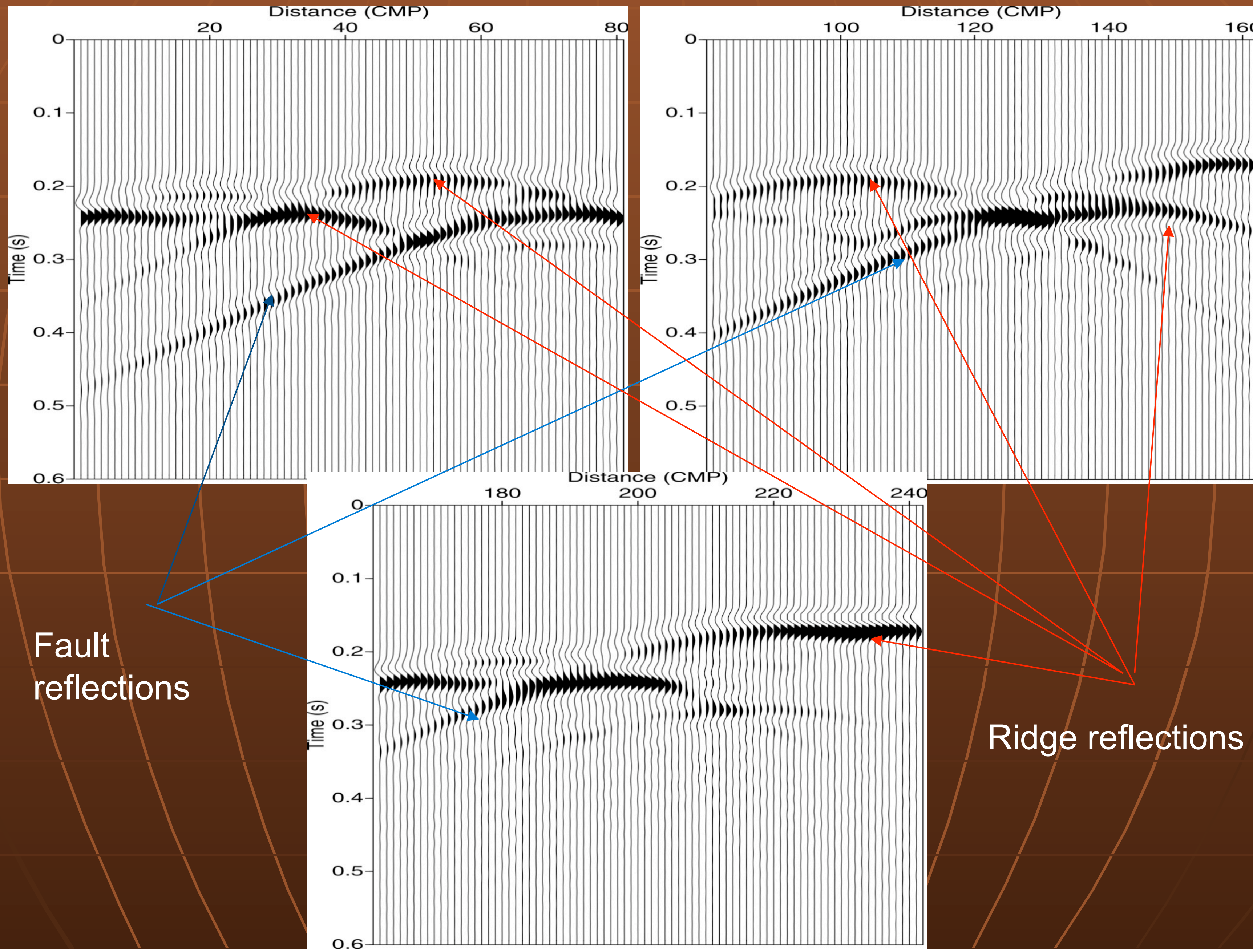


Model size:
625m*625m*100m

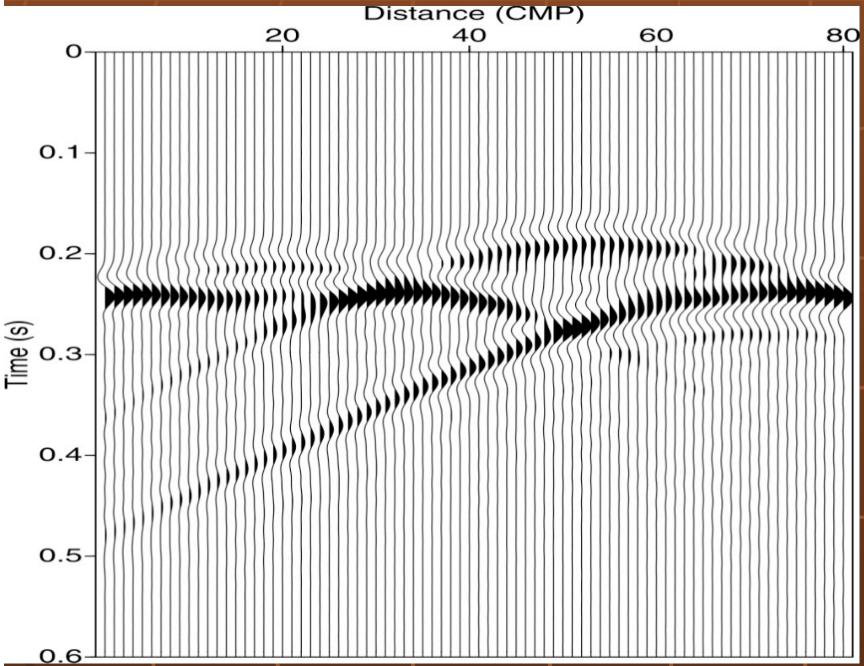
Radius of the hemisphere:
170.8m

W.S. French 1974, Geophysics

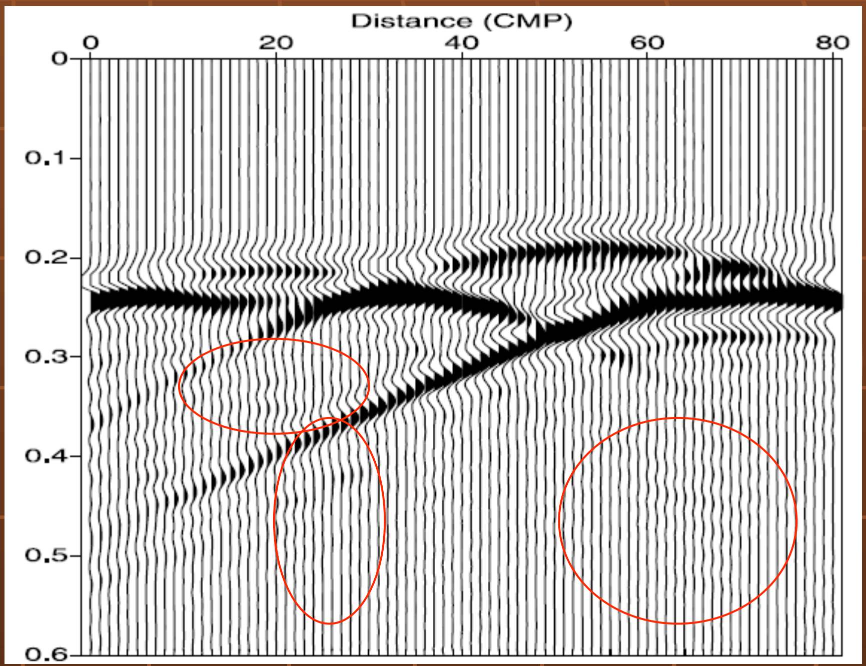
Synthetic seismogram examples



Synthetic seismogram examples



a. no perturbation

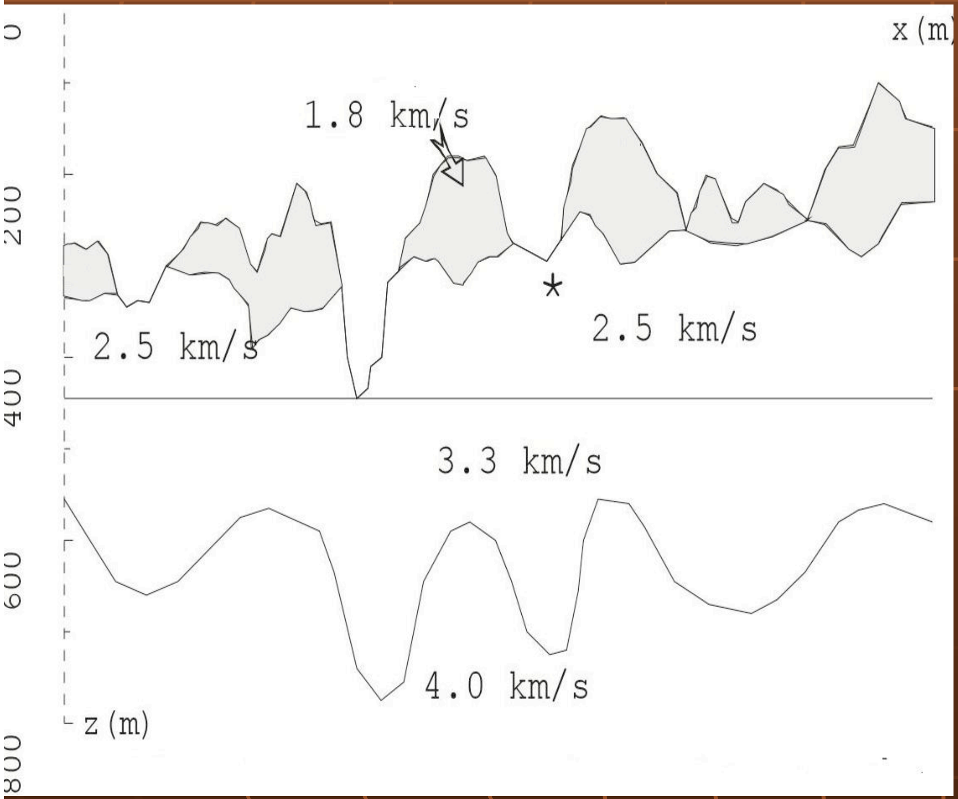


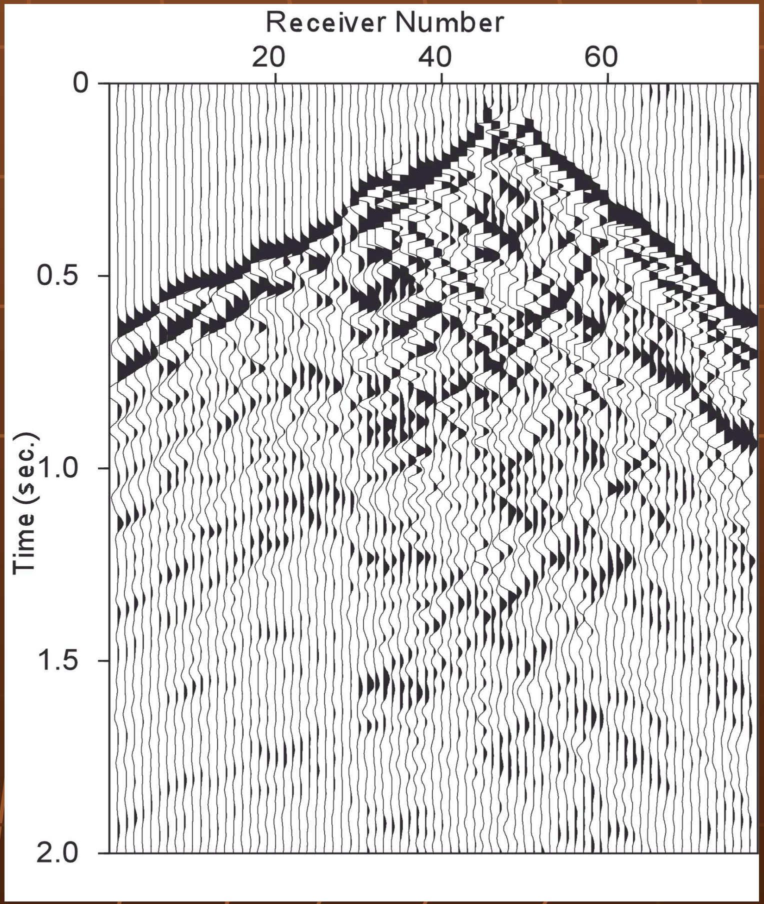
b. δ=0.1

Synthetic seismogram examples

Irregular free surface to simulate land seismic acquisition.

Receivers are along the free surface





Conclusions

Characteristics of the modified Boundary Element Method:

1. Has the advantage of boundary element: requires less data to be calculated.
2. Explicitly use the boundary conditions of continuities across interfaces.
3. Easily adapt to curved free surface and irregular interfaces.
4. Seismic wave scattering by volume heterogeneities is included.
5. While many practical problems need to be solved, this method provides an alternative for seismic forward modeling .

Acknowledgements

- Mrinal Sen
- Robert Tatham
- Kyle Spikes
- Tom Hess
- Students of EDGER forum

Thank you !